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Departamento de
Eletrónica, Telecomunicações e
Informática

David Miguel Esteves
Pereira

Técnicas de Previsão em Redes Celulares

Forecasting Techniques in Cellular Networks

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Dissertação apresentada à Universidade de Aveiro para cumprimento dos requisitos necessários à obtenção do grau de Mestre em Engenharia Eletrónica e Telecomunicações (Mestrado Integrado), realizada sob a orientação científica do Prof. Doutor Aníbal Manuel de Oliveira Duarte, Professor do Departamento de Eletrónica, Telecomunicações e Informática da Universidade de Aveiro.

Dissertation submitted to the University of Aveiro for fulfillment of the requirements to obtain the degree of Master/Doctor in Engenharia Eletrónica e Telecomunicações (Mestrado Integrado), accomplished under the scientific guidance of the Prof. Dr. Aníbal Manuel de Oliveira Duarte, Professor at the Department of Electronics, Telecommunications and Informatics of the University of Aveiro.

Esta dissertação é dedicada ao David, Manuela e Daniela pelo incansável apoio.

Júri / Jury

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Palavras-chave

Dinâmica de Mercado, Técnicas de Previsão, Análise Económica de Cenários de Engenharia, Modelos de Difusão, Análise de Séries Temporais.

Resumo

Esta dissertação apresenta um conjunto de contributos destinados a melhorar a compreensão de alguns aspetos relacionados com o planeamento de redes de telecomunicações. Foram considerados os seguintes aspetos:

- Dinâmicas de Mercado
- Técnicas de Previsão
- Análise Económica de Cenários de Engenharia

Para atingir estes objetivos foram usadas as seguintes ferramentas:

- Modelos de Difusão
- Análise de Séries Temporais
- Construção de cenários de redes de telecomunicações

Keywords

Market dynamics, Forecasting techniques, Economic Analysis of Engineering Scenarios, Diffusion Models, Time Series Analysis.

Abstract

This dissertation attempted to create a better understanding about some of the aspects involved in telecommunications planning. In more specific terms the following aspects were considered:

- Market dynamics
- Forecasting Techniques
- Economic Analysis of Engineering Scenarios

To achieve these goals several tools were used:

- Diffusion Models
- Time Series Analysis
- Construction of telecommunications network scenarios

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Acronyms

BSS – Base Station Subsystem
BSC – Base Station Controller
BTS – Base Transceiver Station
MSC – Mobile Switching Center
2G – Second Generation
GSM – Global Systems for Mobile
SMS – Short Message Service
TDMA – Time Division Multiple Access
FDMA – Frequency Division Multiple Access
GPRS – General Packet Radio Service
3G – Third Generation
UMTS – Universal Mobile Telecommunication System
3GPP – Third Generation Partnership Project
W-CDMA – Wide – Band Code – Division Multiple Access
CDMA – Code Division Multiple Access
HSDPA – High Speed Downlink Project Packet Access
HSUPA – High Speed Uplink Packet Access
4G – Fourth Generation
LTE – Long Term Evolution
OFDM – Orthogonal Frequency Division Multiple Access
VoIP – Voice Over IP
IMT – international Mobile Telecommunications
CoMP – Coordinated Multipoint Transmission/Reception
ICIC – Inter-Cell Interference Coordination
WiMAX – Worldwide Interoperability for Microwave Access
MA – Moving Average
AR – Auto Regressive
SES – Simple Exponential Smoothing
HW – Holt - Winters
ARMA – Auto Regressive Moving Average
ARIMA – Auto Regressive Integrated Moving Average
SSE – Sum Square Errors
MSE – Mean Square Errors
MAPE – Mean Absolute Percentage Errors
ACF – Auto Correlation Function
PACF – Partial Auto Correlation Function
AIC – Akaike Information Criterion
CAPEX – Capital Expenditures
OPEX – Operational Expenditures
NPV – Net Present Value
IRR – Internal Rate of Return
CPU - Central Processing Unit
FIR - Finite Impulse Response
IIR - Infinite Impulse Response
ADSL - Asymmetric Digital Subscriber Line
GPON - Gigabit-capable Passive Optical Networks
DSLAM - Digital Subscriber Line Access Multiplexer
OLT - Optical Line Termination
FTTH - Fiber-to-the-Home

1. Introduction and Motivation

Nowadays, as the result of rapid technological development and regulatory changes, the telecommunications market dynamics has been profoundly affected.

The ability to understand possible life cycle scenarios of a telecommunication product can be an advantage in the planning of its networks, particularly in terms of capacities, investments, resources, marketing and sales. In this context forecasting is of crucial importance.

For these reasons, forecasting in telecommunication planning is now more important than never before [1]

Forecasting is constructing possible futures and preparing to its consequences. It is a continual process, in which all the information available, including historical data and knowledge of any future events, (that might affect the forecasts), are important. Mostly, the forecasting methods depends of what kind of data are available: Whenever it is acceptable to assume that some aspects of the past will continue in the future or that there are numerical data of the past, quantitative forecasting methods can and should be applied. If there is no information from the past, or if it is not relevant enough, qualitative forecasting methods should be used.

There are several quantitative forecasting methods. These methods often arise from very specific areas. Therefore, each method has its own characteristics, which must be taken into account when choosing the method to be used.

It is possible to divide forecasting in two broadly different perspectives.

- Forecasting by inference, when it is reasonable to assume that some patterns and characteristics of the past will be repeated in the future. This leads to forecasting based on time series analogies;
- Forecasting based in analogies with similar cases from the past, this happens, for example, when some new product is launched in the market, for which there is no historical data but there is known how about similar products were adopted. This leads to forecasting based on diffusion process.

Forecasting in the telecommunications market, beyond its usefulness as a tool to help in network dimensioning (of, for example, capacity usage, traffic demand, etc) can also be of great importance in the following situations:

- Market dynamics (e.g., effects of competition between operators);
- Technologies evolution (e.g., substitution and migration of technologies).

The possibility of predicting these interactions certainly brings some advantages to market players.

My motivation, as a newcomer on the area of the telecommunications networks, management and planning, was to study the possible combination of various forecasting techniques to consider substitution and competition effects between technologies and operators respectively.

1.1 Objectives

The main objectives of this work are the following:

- Familiarization with the different short- and long-term forecasting techniques;
- Understand the models of substitution and competition and create analogies with the universe of telecommunications;
- Combine different forecasting approaches and test them with examples from the telecommunications area;
- Apply forecasting techniques in the study of possible techno-economic scenarios associated with the introduction of a new operator in a competitive market.

1.2 Organization of the Dissertation

Besides this introductory chapter, this dissertation is organized in 7 main chapters:

- Chapter 2: Mobile networks Overview, introduces the organization of the cellular network, and the main technologies;
- Chapter 3: Forecasting in New Telecommunications Products, studies the classical methods used for long term forecasts. Start to introduce some important models, and present the logistic model used on this dissertation as well as the substitution and competition models;
- Chapter 4: Forecasting with historical data, studies the classical methods used for forecasting. Starts to introduce some important concepts for a better understand of the chapter and then follow a structure of the type, theoretical study for each model. Besides that, it also presents several concepts and techniques used to evaluate and support the methods;
- Chapter 5: Combining different methodologies of forecasting, on this chapter it was tried to attempt the combination of the long-term and short-term methodologies to predict the patterns of the use of a cellular network;
- Chapter 6: Techno-Economic Analysis, it was elaborated a case study where two telecommunications operators co-exist and it was studied the impact, in terms of cash – balances, that one as on the other, based in some assumptions;
- Chapter 7: Conclusion, it was presented the main results of this dissertation, the balance of the work and suggestions for future work.

2. Mobile Networks Overview

The importance of mobility has led to the great development of mobile telephone networks and subsequently to mobile data networks. There is a need to provide services anywhere, at any time or in any circumstance. Mobile networks allow a user to perform a moving operation.

Cellular networks use the principle of channel reuse (code, frequency, etc.). [2] In a cellular network, the coverage area is divided into regions called cells, so that the transmitted power is low and the available frequencies can be reused.

When connected, these cells cover large geographical areas, provide services to many portable receivers (mobile phones, tabs, Pcs, etc.) and facilitate, through appropriate mechanisms, the mobility of the user.

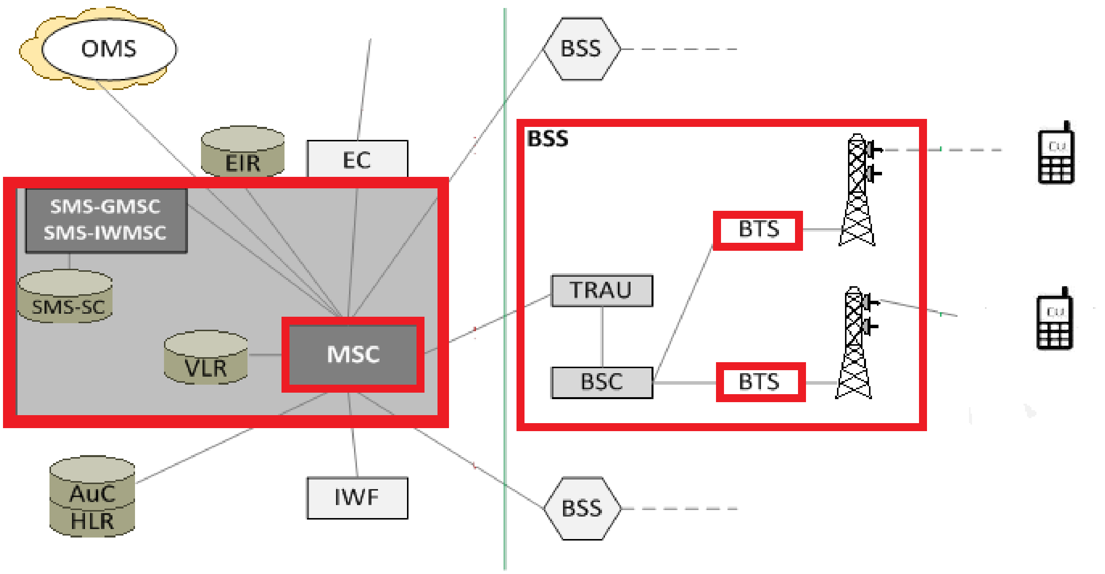

Figure 2-1: Typical Structure of a Cellular Network [3]

From the previous figure, we can observe the typical structure of a GSM network.

The basic elements of a cellular network are [4]:

- Mobile Terminal - Provides the user interface to access the services provided by the operator
- BSS (Base Station Subsystem) - Corresponds to the access network of a GSM system and provides the user with the possibility of switching voice or data. Its nodes interconnect the BTS and the BSC.
- BSC (Base Station Controller) - Controls and manages the BTSs, ensuring the quality and power transmitted by the BTS and allocating a channel for communication during the call.
- BTS (Base Transceiver Station) - interfaces between the Mobile Terminal and the Mobile Switching Center (MSC). These devices define each cell in the network and are composed by the

(i) antenna or antenna array;
(ii) filters,
(iii) duplexers,
(iv) transmitters and receivers,
(v) transmission equipment and physical infrastructure

- MSC (Mobile Switching Center) - The Switching and Control Center corresponds to the core of a cellular network that allows users to validate and process information. It is also responsible for interconnecting to various service providers in addition to base station monitoring and management of hand-off mechanisms that allow user mobility between cells.

2.1 Second Generation Networks (2G) - GSM / GPRS

The most widely used mobile telephone network in Europe is the GSM (Global Systems for Mobile communications) network. It is a telecommunications system that can voice transmission, data services, messaging services and supplementary services such as call forwarding, bus, call and call waiting and short message service: SMS. [5]

The GSM system has switched from analog to digital technology, bringing improvements in security, robustness and reliability.

This technology allows transmission rates of 14.4 kbps.

GSM (Global System for Mobile), also known as the 2G network (second generation network), corresponds to the first digital cellular technology and was the great booster for the massification of the mobile service, driven by the Small Messages Service.

There are several frequency ranges used by the GSM standard, with GSM 900 being the most used. The GSM 900 uses two sets of frequencies in the 900MHz band:

- 890-915MHz for mobile terminal (uplink) transmissions [3]
- 935-969MHz for network transmission (downlink)

Frequency management is done with the combined use of two technologies: Time Division Multiple Access (TDMA) and Frequency Division Multiple Access (FDMA).

The GPRS (General Packet Radio Service) is an evolution of the GSM system, which introduced data transmission with packet switching., working in complementarity with GSM and maintains most of the network equipment.

The GPRS network is interoperable with the large majority of GSM network elements coexisting the two networks in parallel. GSM assures circuit-switched services and GPRS routes packet connections.

2.2 Third Generation Networks (3G) - UMTS / HSPA

3G technology enables operators to offer multiple services simultaneously at higher speeds and support a greater number of voice and data users which is particularly relevant in urban areas. [6]

The ability of 3G networks allows:

- Internet access;
- Mobile TV;
- Video on demand;
- Video conference;
- Geolocation services;
- Telemedicine services;

The UMTS (Universal Mobile Telecommunication System) standard was proposed and developed as an integrated voice and data solution to succeed the 2nd Generation of mobile services (2G), being the first technology to provide the effective use of mobile broadband. The UMTS standardization work was carried out by the Third-Generation Partnership Project (3GPP).

UMTS transmission rates are of the order of 2Mbps, possible through W-CDMA (Wide-Band Code-Division Multiple Access) or CDMA2000 (Code-Division Multiple Access) modulation.

A UMTS system can be implemented based on an existing mobile communications system. Under these circumstances radio equipment can accommodate various technologies such as GSM, GPRS and UMTS simultaneously. This feature makes it easy to migrate from 2G to 3G.

HSPA (High Speed Packet Access) technology emerges as the evolution of the UMTS standard:

- Increases the speed of data transmission through the inclusion of HSDPA (High Speed Downlink Packet Access) channels that allows 14.4Mbps downlink and HSUPA (High Speed Uplink Packet Access) channel for uplink, as an alternative to the UMTS W-CDMA;
- Improves the connection between 3G mobile networks and internet services, allowing broadband access to multiple users served by the same cell.

2.3 Fourth Generation Networks (4G) - LTE / WiMAX

2.3.1 LTE

The great attraction of 4G Networks is the convergence of a wide range of services only available through fixed broadband networks.

LTE (Long Term Evolution) [7] is the norm standardized by version 8 of 3GPP, arising from the need to ensure the competitiveness achieved by 3G technology assumes the following motivations:

- Provide speeds up to 100Mbps Downlink and 50Mbps Uplink;
- Provide optimized for packet switching;
- Reduce latency;
- Reduce the complexity of the network;
- Reduce investment needed for network implementation and operational costs;
- Avoid unnecessary fragmentation of paired (and unpaired) systems with the same operating frequency. The LTE operates at the same frequencies as UMTS with a variable bandwidth of 20Mbps;
- Improve spectral efficiency using Orthogonal Frequency Division Multiple Access (OFDM)

It should be noted that there are two possible implementations:

- Exclusive LTE network - Implemented for data transport. Using voice over IP (VoIP) techniques for voice traffic. This simplified implementation links Nodes B directly to the nuclear network;
- LTE over GSM - Implemented over the existing GSM infrastructure which, through the update of some elements, allows the GSM network to be activated and used to route voice traffic;
- Despite mentioning LTE as a 4G technology, there is some controversy about this because its features do not fully meet the requirements of the IMT Advanced (International Mobile Telecommunications Advanced) protocol. The LTE Advanced, thus emerges as LTE evolution and normalized by 3GPP, fulfilling already all the requirements required to be considered a 4G technology.

From the evolution of LTE to LTE Advanced:

- The aggregation of carriers, allowing the user to receive several frequencies simultaneously;
- Introduction of multipoint transmission / reception mechanisms between CoMP nodes (Coordinated multipoint transmission / reception);
- Advanced Interference Coordination ICIC (Inter-cell interference coordination) in support of heterogeneous networks, minimizing interferences and collisions between cellular networks;

These improvements to LTE Advanced translate into better network throughput than LTE.

2.3.2 WiMAX

WiMAX (Worldwide Interoperability for Microwave Access) [8] is a wireless technology defined by the IEEE 802.16 standard, as an alternative to DSL networks and coaxial cable for internet access.

This technology supports transmission speeds up to 40Mbit / s and has a range of up to 10Km. However, bandwidth is shared by users who are simultaneously connected to the network.

WiMAX networks represent a good solution for mobility and flexibility. However, the fact that they do not allow the provision of services over long distances is a limitation.

However, they represent a solution to cover small remote and rural areas.

3. Forecasting in New Telecommunication Products

When a new telecommunication product is launched there is no historical data about its adoption patterns. In such cases, an analogy with a "similar" technology that has been launched in the past is usually made. In the case of information and communication technology products, diffusion models are the most commonly used.

3.1 Diffusion models

Diffusion models originated from demography and biology [9], and only later they were applied in business economics; their development has been undertaken to try to improve the description of the temporal spread of technologies [10]. The following picture illustrates the aspect of the adoption patterns of several information and communication technology products where diffusion processes were present [11].

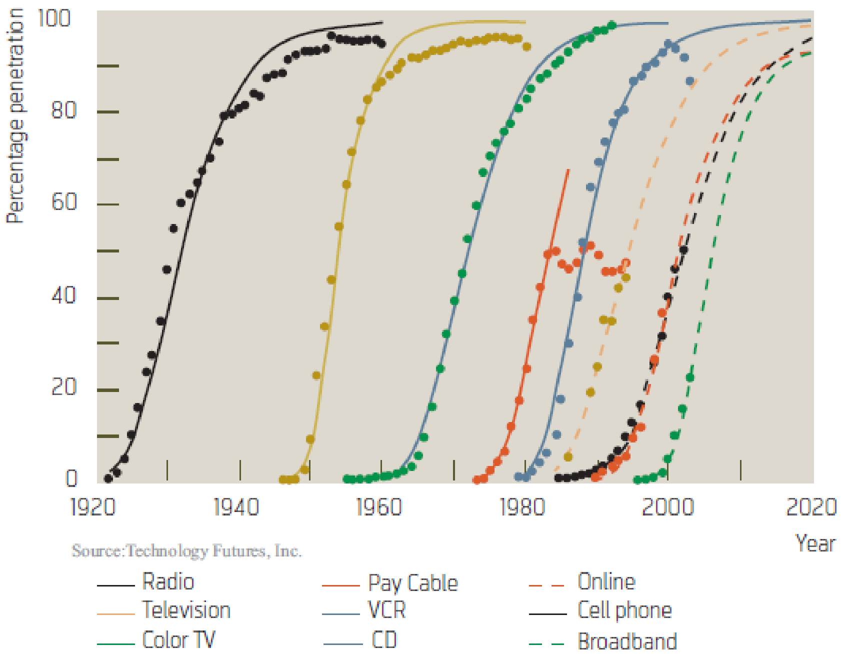

Figure 3-1: Adoption patterns of several technologies in XX century [11].

In literature, forecasting methods based on diffusion processes appear with different designations (Logistic, *Gompertz*, *Fisher-Pry*, *Bass,* etc.). [12]

In this dissertation there will be considered, basically, diffusion models dominated by the effects of innovation and imitation processes [1]:

[1] In addition to the indicated references over the text, the overall contents of this section has been influenced by reference [12].

- Innovation: this macro process translates the effect of something new that arises and that attracts the attention of a user and leads him to want to use this product and incorporates effects such as utility, price and, in some way, design.

- Imitation: even if the emergence of a new product has not immediately attracted the attention of a user, the fact that other people adhere to its use leads to that user is drawn to experience also using. This macro factor incorporates mainly the effects of marketing although other effects such as design can also be reflected there.

These processes take some time to develop and as the number of “catching” users (imitation effect) and the impact of product use (innovation effect) grow, the number of users will grow faster. The tendency will begin to slow down from a point where the amount of “contagious” potentials is already getting scarce and from here the process of “contagion” becomes increasingly difficult and ends up even cancelling itself when the entire “contagious” population is infected.

What has been said can be modelled by the following equation [13]:

$$\frac{dN(t)}{dt} = [\,p + \frac{q}{m}.N(t)].[m - N(t)] \qquad 1$$

Where,

$N(t)$: *represents the accumulated value of the number of adopters until the moment t*
m: *represents the maximum value estimated to be reached by* $N\,(t)$.
q: *contagion coefficient*
(*represents the propensity of users being influenced by other users*).
p: *innovation coefficient*
(*represents the propensity of users being influenced by the novelty of the product*.

The integration of the above equation leads to the following result:

$$N(t) = \frac{m - \dfrac{p(m-N_0)}{p+\frac{q}{m}N_0}e^{-(p+q)t}}{1 + \dfrac{\frac{q}{m}(m-N_0)}{p+\frac{q}{m}N_0}e^{-(p+q)t}} \qquad 2$$

From the above equation, were obtained the following graphical examples:

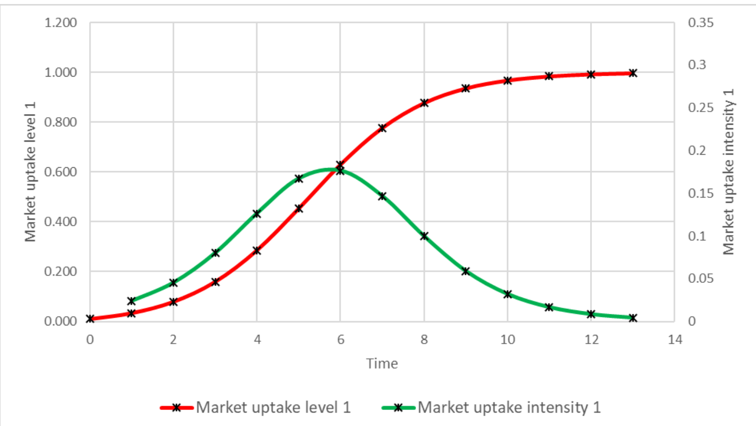

Figure 3-2: Market uptake level and intensity – Imitation dominated

On figure 3-1, parameter $p \to 0$, so the imitation factor dominates the function.

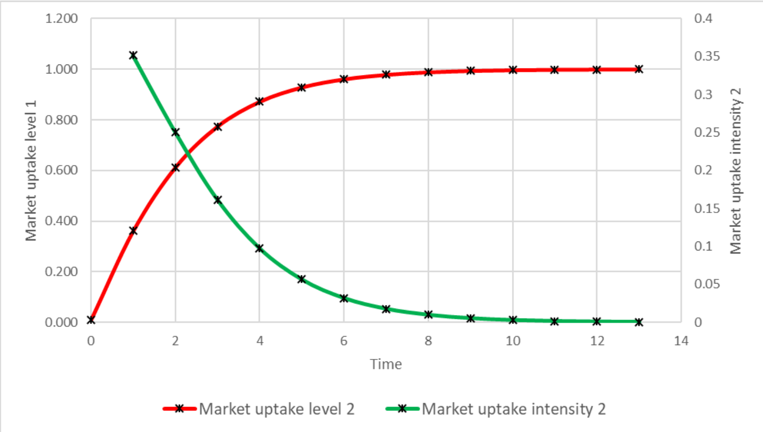

Figure 3-3: Market uptake level and intensity – Innovation dominated

On figure 3-2, parameter $q \to 0$, so the inovation factor dominates the function.

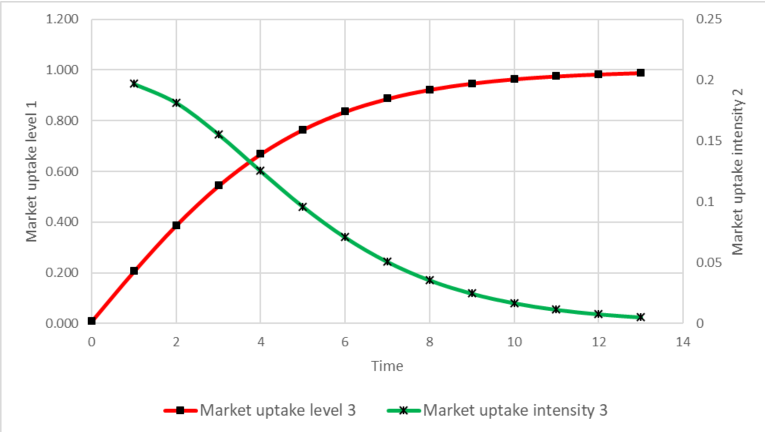

Figure 3-4: Market uptake level and intensity – Mixed Innovation and Imitation scenario

On figure 3-4, parameter $p = q$, so the imitation and innovation factors are both present in the function.

With the above results it was possible understand the mathematical dynamics behind the representation of the diffusion process.

The diffusion model used on this work, is the following present:

$$P(t) = P_i + (P_f - P_i)\frac{1}{1 + \alpha e^{-\beta t}} \qquad 3$$

This model is a variant of the other ones mentioned before, but in this one, are included the parameters that describe the initial and final rate of penetration.

Where:

$P(t) = penetration\ rate\ at\ instant\ \boldsymbol{t}$

$P_i = initial\ rate\ of\ penetration$
$P_f = final\ rate\ of\ penetration$
$\alpha = parameter\ that\ represents\ the\ moment\ in\ which\ the\ market\ starts$
$\beta = parameter\ that\ represents\ the\ speed\ of\ the\ start\ of\ the\ market$

3.2 A Mathematical model of adoption and abandonment innovation and substitution

In a world of constant evolution, technology reflects this same evolution and every year, if not every month, there is news in the most different areas.

Telecommunications is not exception, and in a society where there is an increasing need for more and better services, operators are often required to provide more bandwidth, speed and security in their services to customers.

In a view of these developments, it would be interesting to create a model which would also include the abandonment of technology, if it would be abandoned by customers (by adhering to a newer technology that would provide them with a better service) or replaced by a newer one.

Therefore, a model that incorporates a time window of a technology's life, from its adoption by the clients until its abandonment, will be used. [14]

3.2.1 Bass and Norton model – Model Definition

In the development of this model, the following assumptions were made [15]:

- First, it was assumed that once a given technology/product brings something new, there will be no reversal for the previous technology/product during the period in question. If, for example, adopters subscribe to innovation more than once, subscriptions will consist of the number of subscriptions per subscriber rather than the number of subscribers.

- Second, it was assumed that the average rate of consumption per period approaches a constant. More specifically, if the subscription rate is distributed independently and identically with a constant global average rate. (This rate remains constant because, as the number of subscriptions increases, the change in the average number of subscriptions will be insignificant, due to the introduction of a new subscription).

One can also assume that the number of subscriptions of a given technology changes over time. Technology may have been invented to enter a specific market or to replace an existing one. It may also have purposes that had not been considered until it existed. Finally, a technology may be better than what is currently available.

It is easily observable that the subscription of some generations of technological innovations grows in numbers, explosively. However, as fast as its growth, each technology should have an upper limit in its use, and it is assumed that this limit is constant. It is also assumed that the growth rate of subscriptions of a given technology is related to the number of applications already existing and the number of applications to be realized.

The fact that the average rate of consumption per period is considered to approximate a constant, implying that the growth of technologies constitutes all the variability in the process over time, introduces a significant simplification in the model.

Denote by M the upper limit of the number of subscriptions for which innovation fits. Denote by r the rate at which a given technology consumes foreign interest.

These two variables, assumed constant, multiply to produce the upper limit of subscriptions by time interval, defined by m.

The only change, compared to the one discussed earlier, is the number of subscribers. The process of introducing a new technology into a given market is a process that involves the diffusion of knowledge about the characteristics of technology. The specific functional form chosen to represent this process, in this work, is proposed by the following equation:

$$F(t) = p_i + (p_f - p_i)\frac{1}{1 + \alpha e^{-\beta t}} \tag{4}$$

Thus, in the case of a single generation without successor, the subscriptions would be represented by:

$$S(t) = mF(t) \tag{5}$$

Subscriptions would be proportional to the cumulative (S-shaped) distribution function of the adoption rate. The first differences in subscriptions would be proportional to the rate of adoption.

This model addresses a few generations of innovations. Each generation is introduced to the market before its antecedent has been fully diffused into its potential market. The new generation will get subscriptions for:

- bring an innovation in relation to the previous technology;
- can stimulate interest from new subscribers who were not interested in their antecedent.

In view of the above assumptions, a new notation can be introduced. Let's index the various generations of a technology/product.

Thus, it is possible to denote by S_i the waves of the ith technological generation, and in the case of two generations, it can be write, for the first technological generation:

$$S_1(t) = F_1(t)m_1 - F_2(t-\tau_2)F_1(t)m_1 = F_1(t)m_1[1 - F_2(t-\tau_2)] \quad for\ t > 0$$

And to the second:

$$S_2(t) = F_2(t-\tau_2)[m_2 + F_1(t)m_1] \qquad for\ t > \tau_2$$

Where $S_i(t)$ refers to the subscriptions of a given technology in the ith generation at time t, m_1 refers to the potential for the first generation, m_2 refers to the potential to the potential served exclusively by the second generation.

Note that τ_2 is the time instant at which the second generation is introduced, $F_2(t-\tau_2) = 0$ for $t < \tau_2$.

3.2.2 Model

As has seen previously the process of adopting a given technology can be described by a logistic curve.
Then, let

$$F_i(t) = p_{i_i} + (p_{f_i} - p_{i_i}) \frac{1}{1 + \alpha_i \cdot e^{-\beta_i \cdot t}}$$

be the logistic function that defines the process of adopting a given technology.
It can be assumed that if a given technology has no successor, it will reach a market share determined by,

$$S_i(t) = m_i F_i(t)$$

In the case of dealing with several generations of technological innovation, should take two aspects into account:

- the new generation can keep legacy users of its predecessors – Substitution;
- the new generation can capture the interest of new users that its predecessors have not been able to achieve – Innovation;

Let i be the index of the technological generation, then, S_i is the i-th generation of technology $F(t)$:

$$S_1(t) = F_1(t)m_1 - F_2(t-\tau_2)F_1(t)m_1 = F_1(t)m_1[1 - F_2(t-\tau_2)] \quad for\ t > 0$$

And to the second:

$$S_2(t) = F_2(t-\tau_2)[m_2 + F_1(t)m_1] \quad for\ t > \tau_2$$

It should be noted that the second generation inherits the market share of its previous market (substitution), and has the potential to be able to capture a certain market share m_2 (innovation).

Therefore, it can be said that this model represents simultaneously phenomena of adoption, substitution, abandonment and innovation.

Assume, then, that there are three technological generations of the same technology temporarily out of phase with each other and that their parameters are all the same.

$$F_1(t) = F_2(t) = F_3(t)$$

The model for the three generations is given by,

$$S_1(t) = F_1(t) \cdot [1 - F_2(t-\tau_2)]$$

$$S_2(t) = F_2(t-\tau_2) \cdot [1 + F_1(t)] \cdot [1 - F_3(t-\tau_3)] \qquad 6$$

$$S_3(t) = F_3(t-\tau_3) \cdot [1 + F_2(t-\tau_2) \cdot [1 + F_1(t)]]$$

Where,

$$F_i(t) = p_{i_i} + (p_{f_i} - p_{i_i}) \frac{1}{1 + \alpha_i \cdot e^{-\beta_i \cdot t}}$$

Example:

	1^{th} *Generation*	2^{nd} *Generation*	3^{rd} *Generation*
p_{i_i}	0.01	0.01	0.01
p_{f_i}	1	1	1
α_i	100	100	100
β_i	0.25	0.25	0.25
m_i	1	1	1
τ_i	0	40	80

Table 1: Values of parameters of Bass Model - Example

The graphical result is the following,

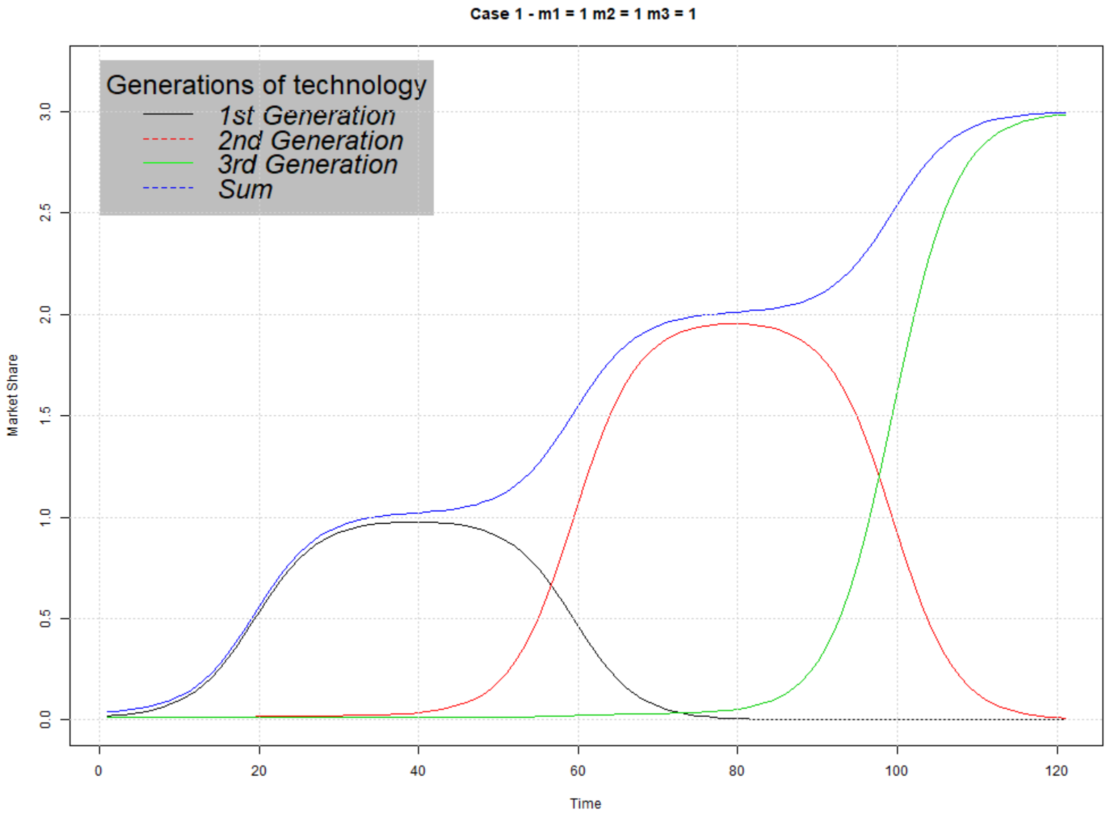

Figure 3-5: Bass Model - Successive generations – case 1

In the above graphic, is possible to see the various technological generations in which is present the various phenomena of adoption, abandonment, substitution and innovation. Is possible to see too, that the last generation has more potential to conquer the market than its previous one.

Considering all the facts mentioned before and if the total market is 1, the following condition must be imposed:

$$0 < \sum m_i \leq 1$$

The m is the parameter that defines the market potential that a certain technological generation can reach, considering that the last generation (the most recent) inherits the market potential of its previous ones. When $m = 0$, it means that the successor technology generation will only replace its previous one and has no innovation effects.

If $m_1 = m_2 = m_3 = 0.33$, the previous condition is satisfied and the graphical aspect is as follows:

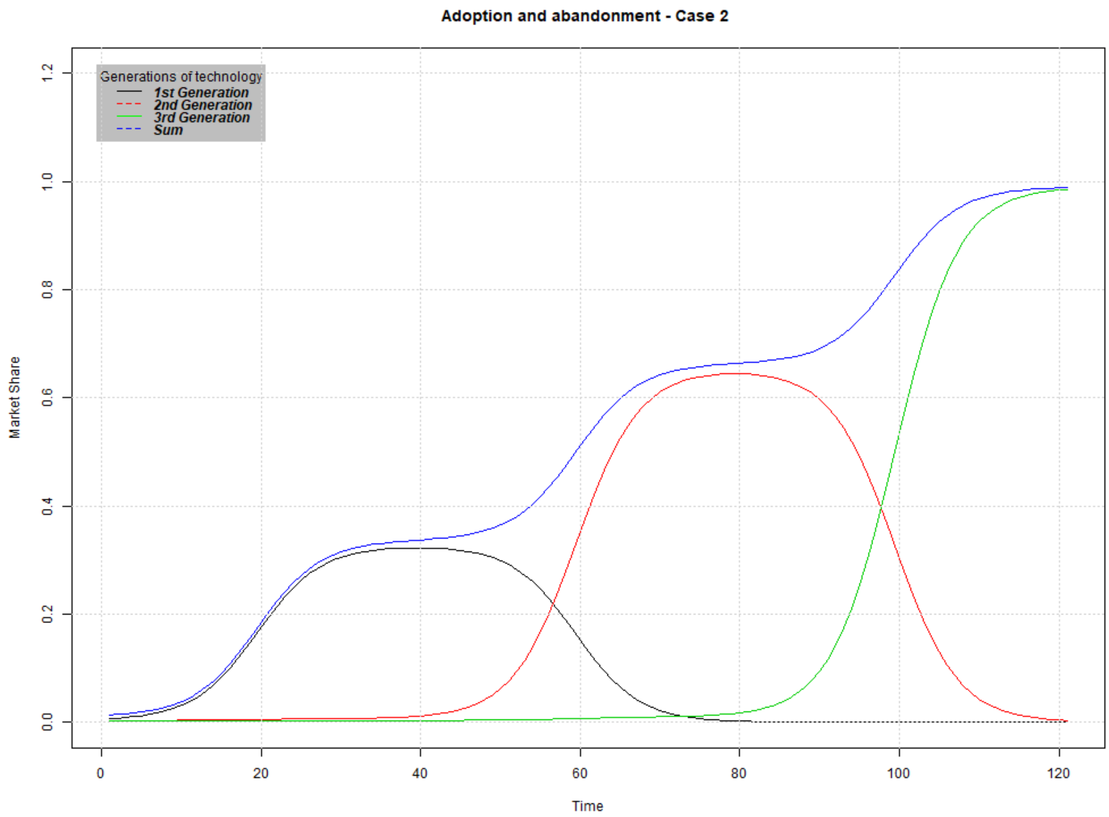

Figure 3-6: Bass Model - Successive Generations - case 2

Assuming now that the first technological generation can reach the total market then it should be considered $m_1 = 1$, $m_2 = m_3 = 0$.

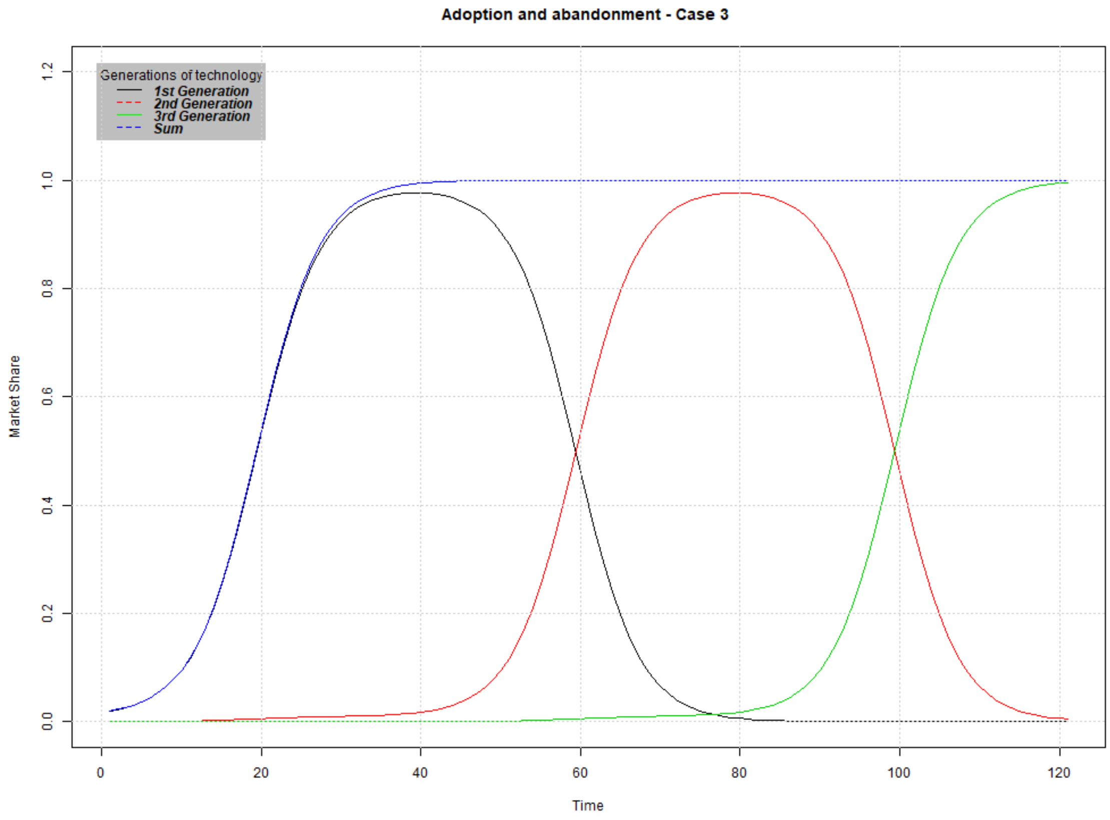

Figure 3-7: Bass Model - Successive Generations - case 3

The market has already been fully won by the first generation and therefore the following only have a substitution effect.

Finally, if the various generations can have different adoption rates and that each of them can reach different market shares, that is, some may or may not introduce an innovation in relation to their previous one.
Let $p_{i_i} = 0.01$ and $p_{f_i} = 1$, in all generations. And considering the following parameters:

$\alpha_1 = 100$	$\beta_1 = 0.1$	$m_1 = 0.5$	
$\alpha_2 = 250$	$\beta_2 = 0.3$	$m_2 = 0.3$	$\tau_2 = 40$
$\alpha_3 = 400$	$\beta_3 = 0.5$	$m_3 = 0.2$	$\tau_3 = 80$
$\alpha_4 = 500$	$\beta_4 = 0.6$	$m_4 = 0$	$\tau_4 = 120$

Table 2: Values of parameters of Bass model - Example

The graphical aspect is the following:

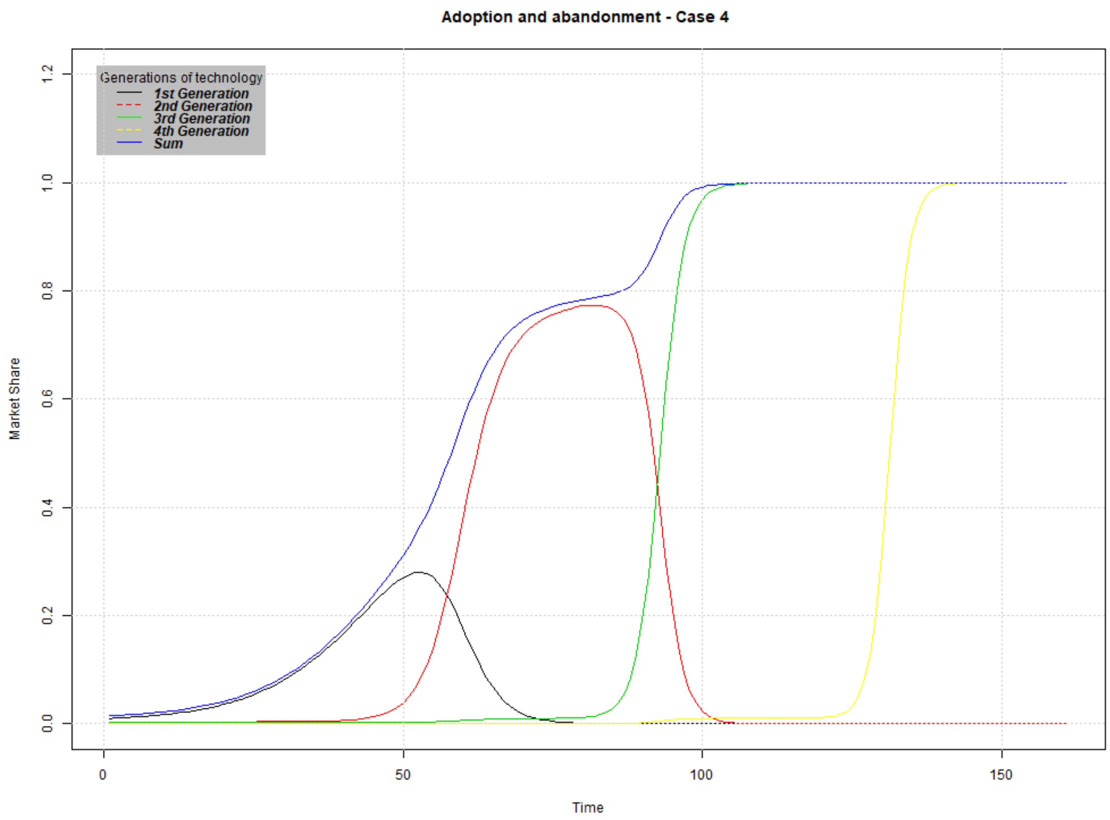

Figure 3-8: Bass Model - Successive Generations - case 4

The above graphic shows that the discussed model can represent all the phenomena's that a given technology may be disposed.

3.3 A mathematical model for competition – The Lotka and Volterra curves

3.3.1 Lotka and Volterra competition model

Lotka-Volterra model is the simplest model of predator-prey interactions. The model was developed independently by Lotka (1925) and Volterra (1926) [12]. From a mathematical point of view the Lotka – Volterra equations are a pair of first order, nonlinear, differential equations. These equations are usually used to describe the population dynamics of biological systems, especially when two species interact, one as prey and the other as predator.

The competition for resources, predators-prey, host-parasites and other types of interactions integrate species into a dynamic population system.

Type of interaction	Expected Outcome
Pure competition	Decrease in both species
Predator / Prey	Predator increases, prey decreases
Mutualism	Increase in both species
Parasitism	Parasite increases, host decreases
Commensalism	One increases, the other not affected

Table 3: Outcome of interactions [16]

Analyzing the table above, the competition for limited resources between two species has a negative effect on the population of both species. In predation and parasitism, the abundance of predators and parasites is expected to increase at the expense of prey and host, since predators feed on preys and parasites obtain benefits from the host. In commensalism, one species is benefited, while the other is not harmed. In mutualism, two species help each other and both species benefit. [16]

Despite Lotka-Volterra model is well known concept in ecological science for years, its application in business is relatively new. Some authors have been tried, over the years adapt this model to economic areas [13].

The work that will be presented in this subsection is a demonstration that it is possible to apply the Lotka Volterra model to the telecommunications universe to predict situations of competition between different actors. This work has a strong inspiration in works developed by other authors in ecology because they help strongly in the understanding of some assumptions to consider.

So, if we have a dynamical market, in which several operators interact, continuously, by the bigger market share or a situation where a new technology interacts with an existing one, the Lotka – Volterra model can be applied because the behaviour among the actors in these situations is similar that of two species a predator and another prey. [17]

This consists in the study of the market shares of operators or technologies, adopting approaches from evolutionary theory of population biology and population dynamics.

3.3.2 Population dynamics

Population dynamics is the study of short or long-term changes in the numbers, individual weights and age composition of individuals in one or several populations, and biological and environmental processes influencing those changes.
The corresponding population modelling is an application of statistical models to the study of these changes in populations because of interactions of organisms with the physical environment, with individuals of their own species (intra-species competition), and with organisms of other species (interspecies competition).
Finally, one of the most important questions population modelling seeks to answer is if interacting species can coexist or not and what are the major factors that affect coexistence. [18]

3.3.3 Example of application

To show a possible implementation of the Lotka and Volterra model, it was considered the approach carried out by the Greeks *Michalakelis*, *Christodoulos*, *Varoutas* and *Sphicopoulos*, in the work *"Dynamic estimation of markets exhibiting a prey predator behaviour"* [18]. These researchers tried to adapt the *Lotka and Volterra* model to the telecommunications market. [15] They verified that in certain situations the different players of this market behave in a similar way to the different species in the ecology. Thus, they define a model based on the *Lotka and Volterra* curves, which is presented below.

$$\frac{dx(t)}{dt} = r \cdot x(t) \cdot \left(1 - \frac{x(t)}{K}\right) - b \cdot x(t) \cdot y(t)$$

7

$$\frac{dy(t)}{dt} = -c \cdot y(t) + d \cdot x(t) \cdot y(t)$$

In which:

$\frac{dx(t)}{dt} = rate\ of\ population\ change\ for\ the\ prey$
$\frac{dy(t)}{dt} =\ rate\ of\ population\ change\ for\ the\ predator$
$r = Growth\ rate\ of\ prey$
$K = Carrying\ capacity\ (the\ maximum\ potential\ of\ the\ market)$
$b = Constant\ used\ to\ capture\ the\ prey'sextinction\ rate\ as\ a\ result\ of\ interaction\ with$
$predator$
$c = Constant\ that\ captures\ the\ prey'sdeath\ rate$
$d = Measures\ the\ predators\ growth\ as\ a\ result\ of\ interaction\ with\ the\ prey$

These constants are all positives.

3.3.4 Case study description

The previously exposed methodology was applied to the case immediately after liberalization of the telecommunications market, since the alternative operators that appeared prayed on the incumbent operator's market share, acting as predators. Therefore, the incumbent operator's market share started to decrease, while on the same time the alternatives market shares increased. [18]

3.3.5 Estimation of the model parameters

To evaluate the effectiveness of the proposed model, the parameters of the equations 1 and 2, should first be estimated. Such estimations are usually achieved by making reasonable assumptions based on the available data, but in the present methodology heuristic methods are employed by the means of genetic algorithms. Genetic algorithms are applied over a dataset to "train" the system, in terms of estimating the model parameters. [18]

3.3.6 Case study results

After the application of the genetic algorithm it was reached the following values of the parameters of equations 4 and 5:

$$\frac{dx(t)}{dt} = 1.76 \cdot x(t) \cdot \left(1 - \frac{x(t)}{0.9}\right) - 1.75 \cdot x(t) \cdot y(t)$$

$$\frac{dy(t)}{dt} = -0.11 \cdot y(t) + 0.23 \cdot x(t) \cdot y(t)$$

And the graphical result obtained was:

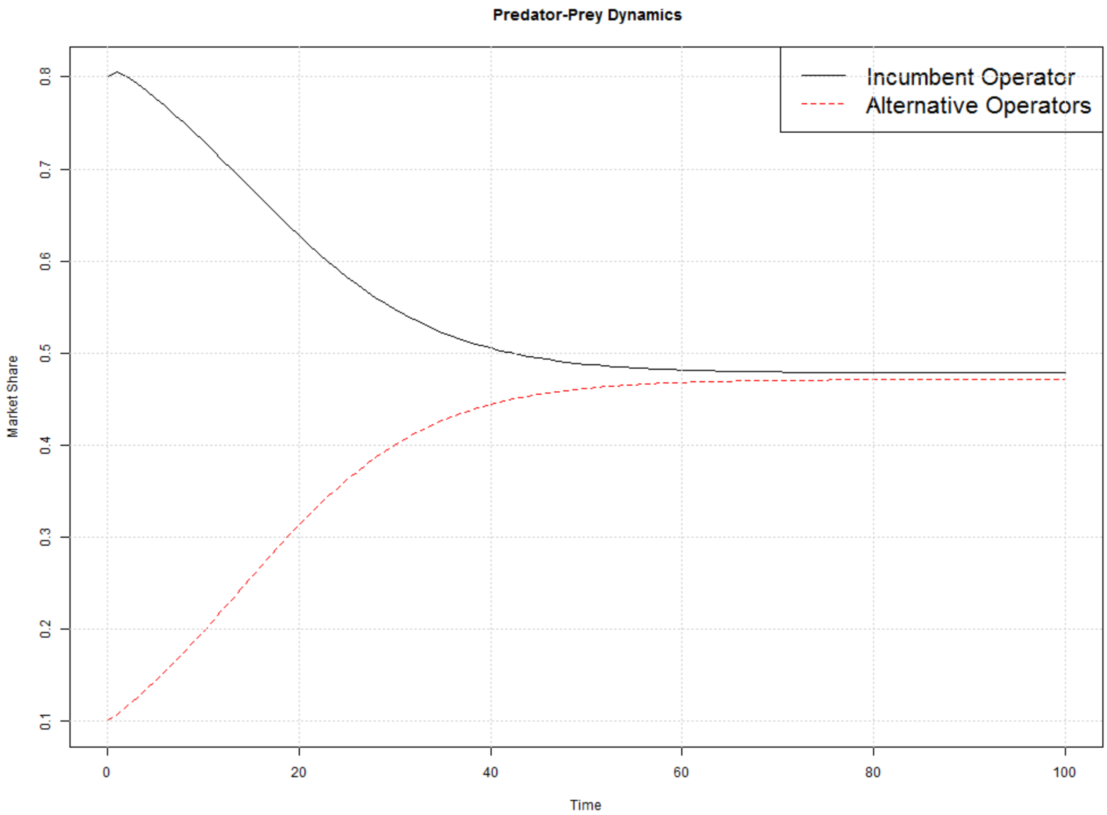

Figure 3-9: Predator-Prey Dynamics.

In the figure, is possible see, to a certain extent, that the market share of the incumbent operator decreases as alternative operators grow. Later, they are in a situation of stability, in which they both share equally the available market.

However, this model developed by the authors referred above is too simple. They do not consider the hypothesis of the two species have been equal in terms of dynamics (the equations that represent each species are different). So, it is possible understand that is not a model who could be apply to different cases.

3.3.7 Lotka Volterra models

To fulling modelling the relation and effect between two different operators in the telecommunication market, the Lotka-Volterra (LV) models are a reliable approach. Since this model was initially developed in the context of the study of biological species interactions, following will be detailed and explained the fundaments of the model and the necessary modifications to fit the telecommunications market requirements.

3.3.7.1 Predator-prey interactions

The development of LV model was done considering the predator-prey interactions and its influences between two species. That lead to the following:

$$\frac{dH}{dt} = rH - aHP$$
$$\frac{dP}{dt} = bHP - mP \qquad (8)$$

In which:

- $H = Prey's\ density$
- $P = Predator's\ \ density$
- $r = Intrinsic\ rate\ of\ prey\ population\ increase$
- $a = Predation\ rate\ coefficient$
- $b = Reproduction\ rate\ of\ predators\ per\ 1\ prey\ eaten$
- $m = Predator\ mortality\ rate$

To solve these differential equations there are two major approaches: analytical or numerical. The analytical methods are complex and require advanced mathematical skills and many differential equations have no analytical solution. The numerical methods are less complex and more commons (however, they could have problems with convergence). Using the numerical approach with a discrete analysis, the above equations are equivalents to the following ones:

$$H(t+1) = H(t) + rH(t) - aH(t)P(t)$$
$$P(t+1) = P(t) + bH(t)P(t) - mP(t) \qquad (9)$$

The line graph bellow represents the populations of two species in a predator-prey system of the Lotka-Volterra model over a period of time.

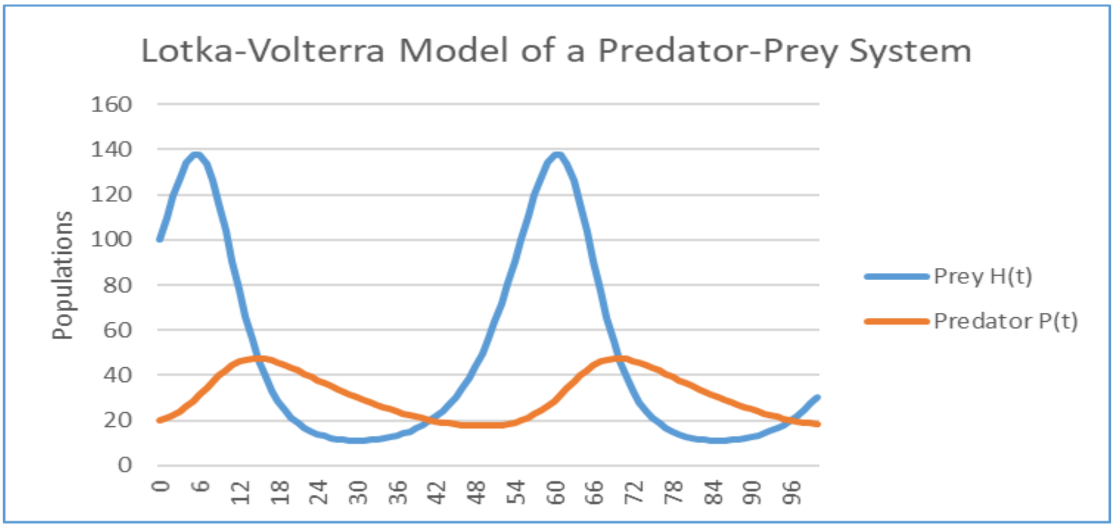

Figure 3-10: Lotka-Volterra Model of Predator-Prey System

In general, both prey and predator populations increase. When it reaches the maximum population, the prey begins to decrease rapidly. The population of the predator also begins to decline, because their way of subsistence, the prey, is diminishing. This behaviour is repeated over time [19].

This LV model is not very realistic because it does not consider competition between prey and predators. Thus, the prey population grows infinitely without resource limits. Predators do not have saturation: their rate of consumption is infinite. The rate of prey consumption is proportional to the prey density. The behaviour of this model is natural because it does not show asymptotic stability [19].

3.3.7.2 Functional and numerical response

A more realistic approach in the relationship between two species would be to consider the competition of the species for the available resources. This consideration led to the claim that predation rates increased with the increase in prey population density. This resulted from two effects [19]:

1. When exposed to a higher prey density each predator increased its consumption rate [19];
2. Predator density increases with the increasing prey density [19].

These effects as two kinds of responses of predator population to prey density:

1. The functional response [19];
2. The numerical response [19].

3.3.7.3 Modelling functional response

An improvement, that is very popular among ecologists and called “disc equation”, was proposed by Holling, suggesting a model of functional response. This model represents the principal of time budget in behavioural ecology. Based in this model the predators spend time in two different activities [19]:

1. Preys search [19];
2. Handle the prey: chasing, killing, eating and digesting [19].

In this model the rate of consumption of a predator is limited. For even if the prey population is so large that no time is needed for searching, a predator still needs to spend time on prey treatment (handling) [19].

Total time equals to the sum of time spent on searching and time spent on handling [19]:

$$T = T_{search} + T_{handling}$$

Assume that a predator captured H_a prey during time T. Handling time should be proportional to the number of prey captured [19]:

$$T_{handling} = H_a T_h$$

$T_h = time\ spent\ handling\ the\ prey$ [19].

The capture of a prey is a random process. The predators examine an area a per unit of time and captures all the preys they find there. [19].

Let's consider T_{search} the time spent on searching, aT_{search} the time spent on the examination of the area, and aHT_{search} the preys captured. H is the preys density per unit of area [19]:

$$H_a = aHT_{search} <=> T_{search} = \frac{H_a}{aH}$$

The time budget:

$$T = T_{seach} + T_{handling} = H_a T_h + \frac{H_a}{aH}$$

So the number of attacked prey $\boldsymbol{H_a}$, is given by:

$$H_a = \frac{aHT}{1 + aHT_h}$$

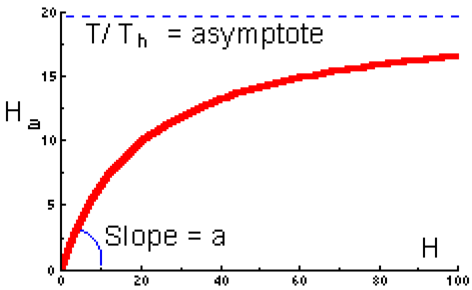

Figure 3-11: Functional response that corresponds to H_a. [19]

In the line graph above is possible observe the number of attacked preys by a predator on various prey's densities. It is possible conclude that when the prey's density is low the most of time of predators is spend searching and when prey density is high the most of the time is spend on handling. When predators increase their search activity because of the increasing of prey's density occurs the functional response. [19].

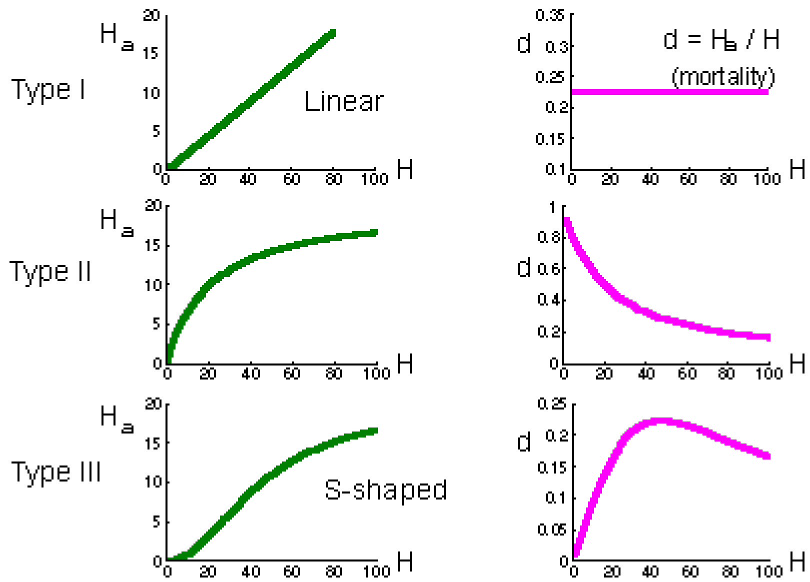

Figure 3-12: The three major types of functional response. [19]

Type III functional response is the only type of functional response for which prey mortality may increase with increasing prey density. But in this case the regulatory effect of predators is limited to the range of prey density, where mortality increases. If the density of the prey exceeds the upper limit of this range, the mortality due to predation begins to decrease, and predation will provoke a positive feedback. Thus the number of prey will be uncontrolled. They will grow in number until other factors stop their reproduction [19].

3.3.7.4 Modelling Numerical Response

Another important consideration is the numerical response, that means that number of predators grows as number of preys increases. But, the numerical response could be the result of two different aspects [19]:

1. When preys are abundant, the rate of predator reproduction increases [19];
2. Attraction of predators to prey aggregations ("aggregational response") [19].

If predation rate increases, reproduction of predators increase and their mortality rate decreases. [19].
The predator's numerical response model is like a conversion of preys in new predators. That happens because this model assumes that reproduction of predators is proportional to the preys consumed. The aggregation of predators to prey density is commonly known "aggregational response". [19].

3.3.7.5 Predator-Prey Model with Functional and Numerical Responses

Joining the approaches detailed above, it is possible to build a full model of predator-prey that includes both the functional and numerical responses [19].
Starting with prey population, predation rate is simulated using "disc equation" of functional response [19]:

$$H_a = \frac{aHT}{1 + aHT_h}$$

The rate of prey consumption by all predators per unit time is given by,

$$\frac{H_{aP}}{T} = \frac{aHP}{1 + aHT_h}$$

The equation of prey population dynamics is:

$$\frac{dH}{dt} = r_h H \left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h} \qquad 10$$

In which $K = Carrying\ capacity\ of\ preys$.

In this case is assumed that if there is not predators, prey population grows according to the logistic model [19].

Predators dynamics is modeled by a logistic model with carrying capacity proportional to the number of preys [19]:

$$\frac{dP}{dt} = r_P P \left(1 - \frac{P}{kH}\right) \qquad 11$$

In which $k = Carrying\ capacity\ of\ predators$.

This equation represents the numerical response of predator population to preys density [19].

3.3.7.6 Competition between species

The above results also assume the existence of predatory and prey species that limit the reality of the ecosystem because similar species are the main factor determining the structure of the population communities. The question then arises whether competing species may or may not coexist and what factors determine this coexistence. [19]. A major challenge is prevent the extinction of a certain species; predict potential losses in species composition after introduction of competitors; to reduce competition effects [19].

In the logistic model, population density converges to the carrying capacity K, as it is shown below [19]:

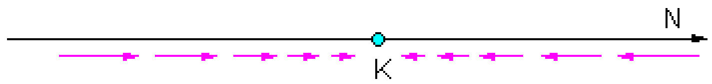

Figure 3-13: Population density converging to the carrying capacity K [19].

Now, it will be introduced the second (competing) species [19]:

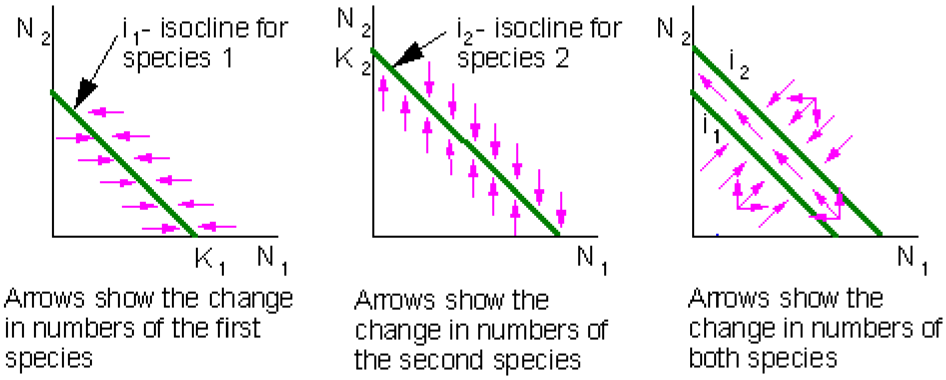

Figure 3-14: The populations densities converges to the carrying capacity K [19].

In this case, both straight lines are parallel and have a slope of 45 degrees. The species that have the highest carrying capacity (K) always wins. Higher carrying capacity means that species can support more agglomeration than other species (for example, due to the more effective search for resources). Competitive exclusion is called K selection because it will always towards increasing [19].

In this example, species 1 becomes extinct because of its competition with species 2. Competitive exclusion principle was first formulated by *Grinnell* (in 1904), who wrote: [19]

"Two species of approximately the same food habits are not likely to remain long evenly balanced in numbers in the same region. One will crowd out the other; the one longest exposed to local conditions, and hence best fitted, though ever slightly, will survive, to the exclusion of any less favoured would-be invader." [19]

If the competing species i.e. use the same resource, the inter-specific competition is the same as the intra-specific competition. Each element competes with all elements of the two populations. It can then be concluded that the growth rate of each population is determined by the sum of the sizes of both. [19]:

$$\frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1 + N_2}{K_1})$$

$$\frac{dN_2}{dt} = r_2 N_2 (1 - \frac{N_1 + N_2}{K_2}) \qquad 12$$

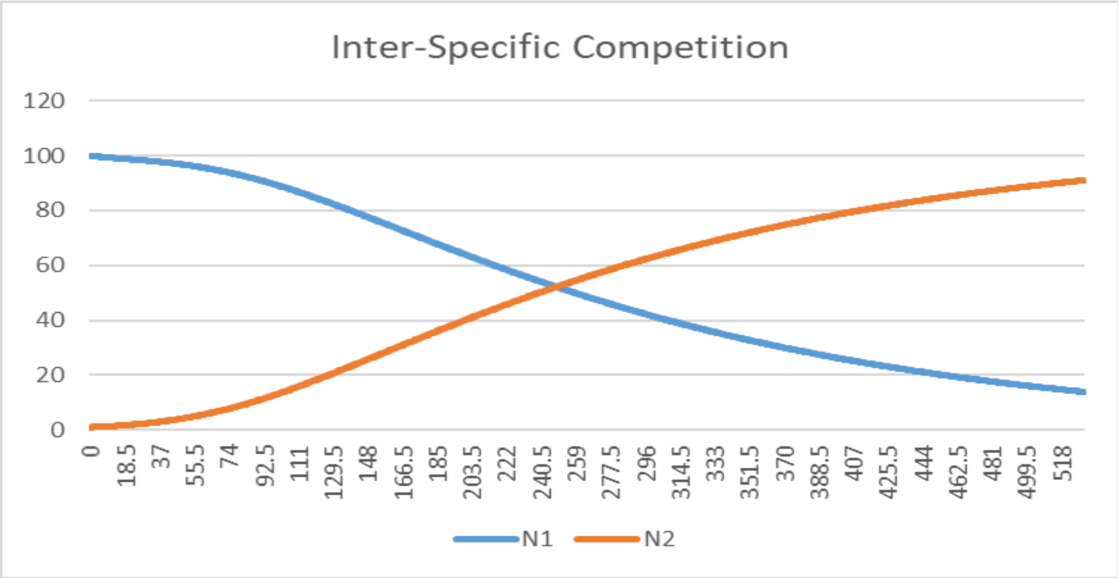

Figure 3-15: Inter-Specific Competition (Lotka-Volterra Model)

When competing species are sufficiently different, intra-specific competition overlaps inter-specific competition. Organisms of other species are not considered "full" competitors. As a result, the number of inter-specific competitors is multiplied by a weight $w_i < 1$ [19]:

$$\frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1 + w_1 N_2}{K_1})$$
$$\frac{dN_2}{dt} = r_2 N_2 (1 - \frac{w_2 N_1 + N_2}{K_2}) \quad 13$$

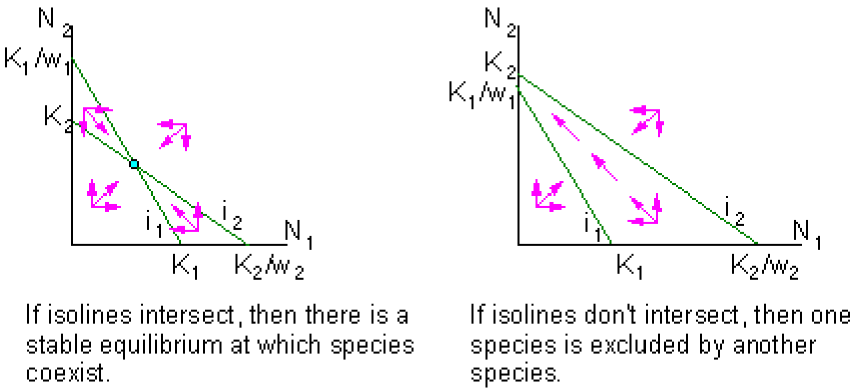

Figure 3-16: The population densities converge to the carrying capacity K with weight. [19]

It is possible that weights $w_i > 1$. This means that organisms of another species are stronger than organisms in the same population. If $w_i > 1$ and the straight lines intersect, then one species will oust the second one from the ecosystem. But what species will be excluded depends of the initial sizes of both populations [16]:

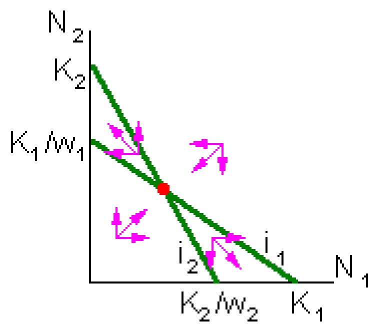

Figure 3-17: The populations densities converge to the carrying capacity K with weight. [19]

There are two areas of attraction where this system has an unstable equilibrium. There are where one of the species oust the other [19]. The coexistence of species occurs if intraspecific competition is superior to interspecific competition. This occurs if competing species use resources differently [19].

3.3.8 Telecommunications analogy

Considering the telecommunications eco-system, some analogies between the behaviour of the ecology players and the telecommunications actors will be explored.

Pure competition

The competition of similar species in ecology could be compared to the competition between two telecommunications operators. The equivalent to “life resource” in ecology is the number of subscribers in telecommunications. The competition instruments are the same, between operators, such as price plans, offer of new services or new technologies, etc.

Thus, let’s assume that the search for prey is the search of an operator for new clients or subscribers. The prey handling is the time spend by an operator to develop new ways to attract new subscribers. That ways could be new plan prices or the launch of new technology or service. At this point it was considered that the players of a certain telecom market act like predators and preys. H_a represents the number of subscribers that an operator got attract.

Taking in account, the above assumptions it is possible making use of the following equations:

$$\frac{dN_1}{dt} = r_1 N_1 (1 - \frac{N_1 + w_{12} N_2}{K_1})$$
$$\frac{dN_2}{dt} = r_2 N_2 (1 - \frac{w_{21} N_1 + N_2}{K_2}) \qquad 14$$

In a discrete analysis:

$$N_1(t+1) = N_1(t) + r_1 N_1(t)(1 - \frac{N_1(t) + w_{12} N_2(t)}{K_1})$$
$$N_2(t+1) = N_2(t) + r_2 N_2(t)(1 - \frac{w_{21} N_1(t) + N_2(t)}{K_2}) \qquad 15$$

In which:

$N_i = Initial\ percentage\ of\ clients\ or\ subscribers\ \ of\ operator\ i$
$r_i =\ growth\ rate\ of\ operator\ i$
$w_{ij} = weight\ of\ the\ effect\ of\ operator\ j\ on\ operator\ i$
$K_i = carrying\ capacity\ of\ operator\ i$

The parameter w_{ij}, represents the effect that a player has on the other. If the $w_{ij} < 1$ then, the both players coexist in the eco-system. But, if $w_{ij} > 1$ one of the players oust the other from the eco-system. The player that will be oust, it will depend on the initial number of clients or subscribers of them.

Let's consider, an eco-system where there are only two players (operators). And let's assume that the both players have equal market shares, and compete for new clients. At certain point one of them adopt some strategy that gives him some advantage against the opponent. If the parameters of the previous equations were accurately estimated, it will be possible represent and predict a similar scenario.

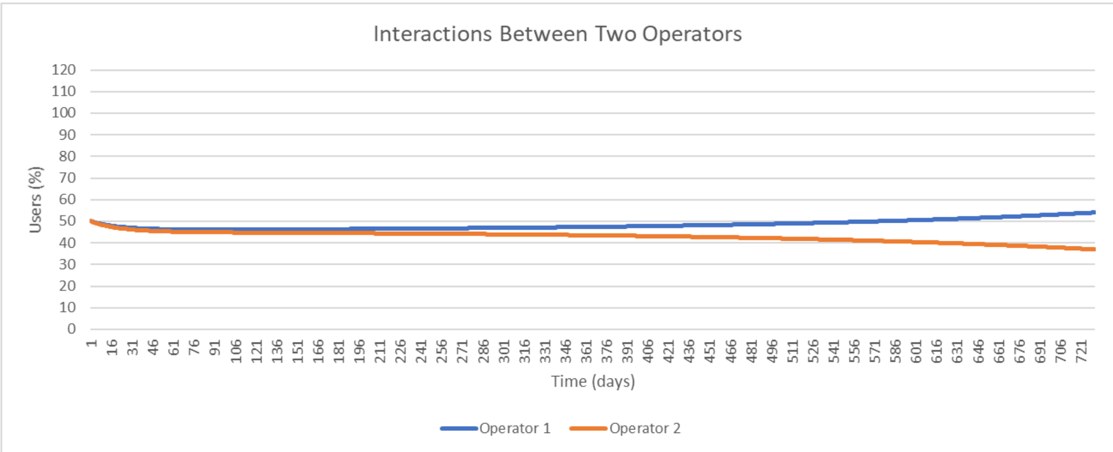

Figure 3-18: Competition with annihilation – Case 1

The line graph above, shows the previously described. In the beginning the two players had the same share of the market, and they harm each other. After the player, one adopts a different strategy it is possible see a little decrease in both shares, but then the market share of the operator 1 starts increase while the share of the operator 2 continues decrease. This is a situation of choice, i.e., there is something in the new strategy that made some clients prefer the player 1 against player 2. At some time, the operator 2 will annihilated from the market.

Let's assume another situation, all the parameters are the same in both operators, but the market share, of operator 2, was 1% bigger than the operator 1. On that case, the operator 2 increases and operator 1 decreases.

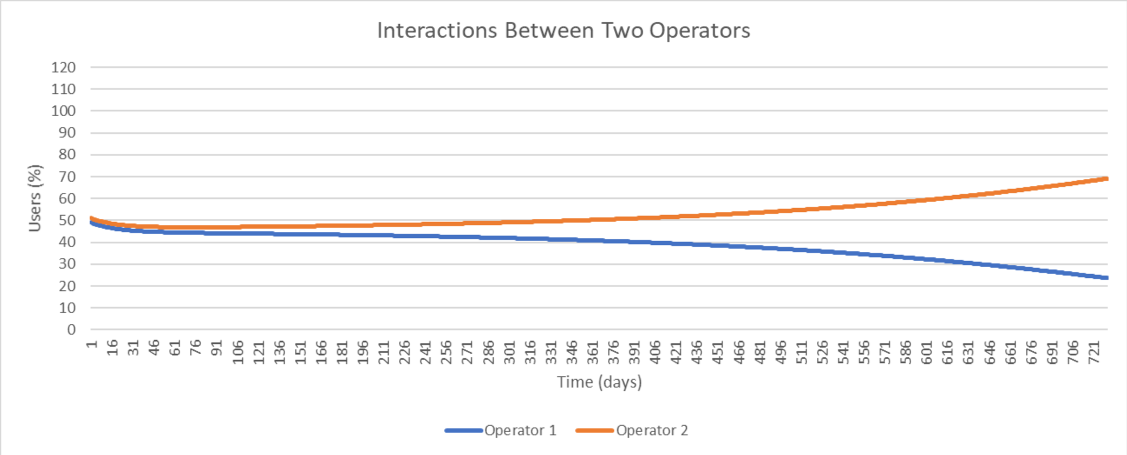

Figure 3-19: Competition with annihilation - Case 2

If the operator 1 had 51% and operator 2 had 49%, the opposite of the previous case, operator 1 increases and operator 2 decreases. As it is shown on the following graph.

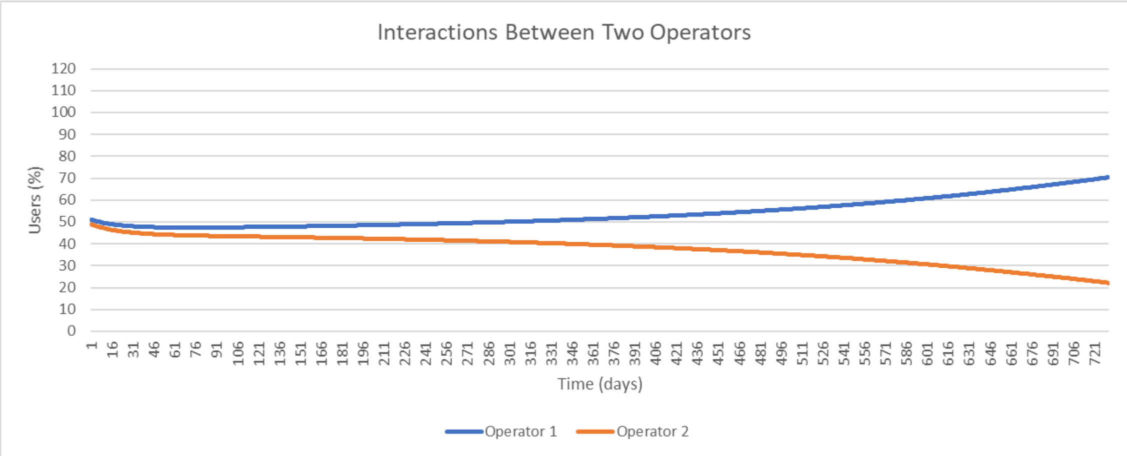

Figure 3-20: Competition with annihilation - Case 3

Coexistence cases:

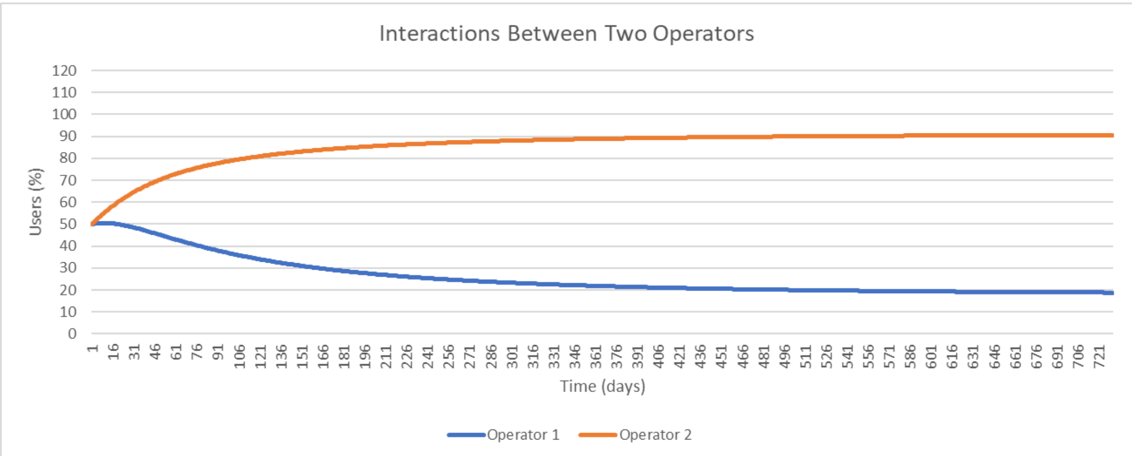

Figure 3-21: Competition without annihilation – Case 1

Here, the two players share the market on the initial phase. Then, the operator 2, get more market share and the operator 1 lost. However, at certain point the situation stabilizes and the two players can coexist in the eco-system.

Let's assume, now, a case, where there is an incumbent operator (operator 1) and an emerging one (operator 2) appears on market. But on that case, the emerging operator, does not have the ability to oust the incumbent operator of the market. The following chart, represents the scenario.

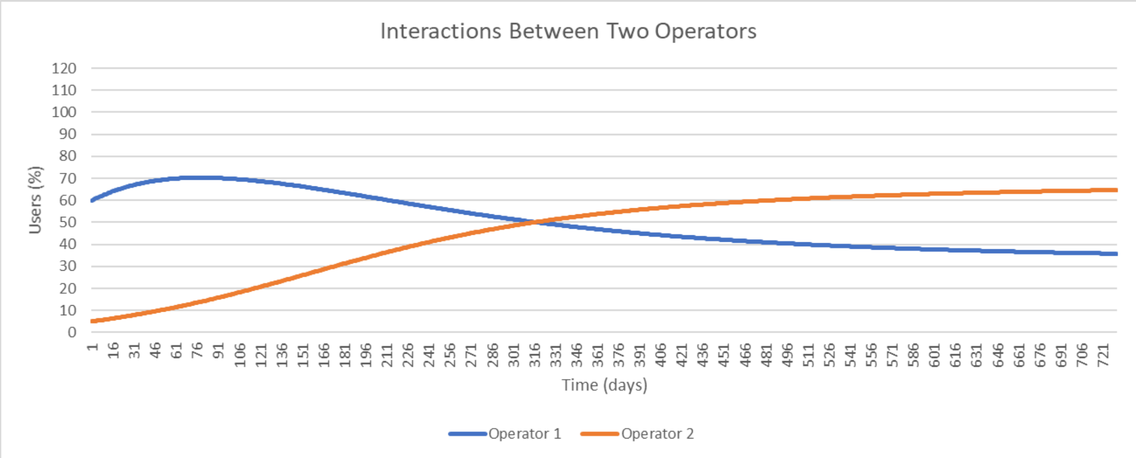

Figure 3-22: Competition without annihilation - Case 2

Other situations can be representing by this model. Such as, competition with market inversion and annihilation. Let's assume that there is an incumbent operator in the market, and a new operator it is launched in the same market, and the strategy of the new one is strong enough to attract all the clients of the incumbent one.

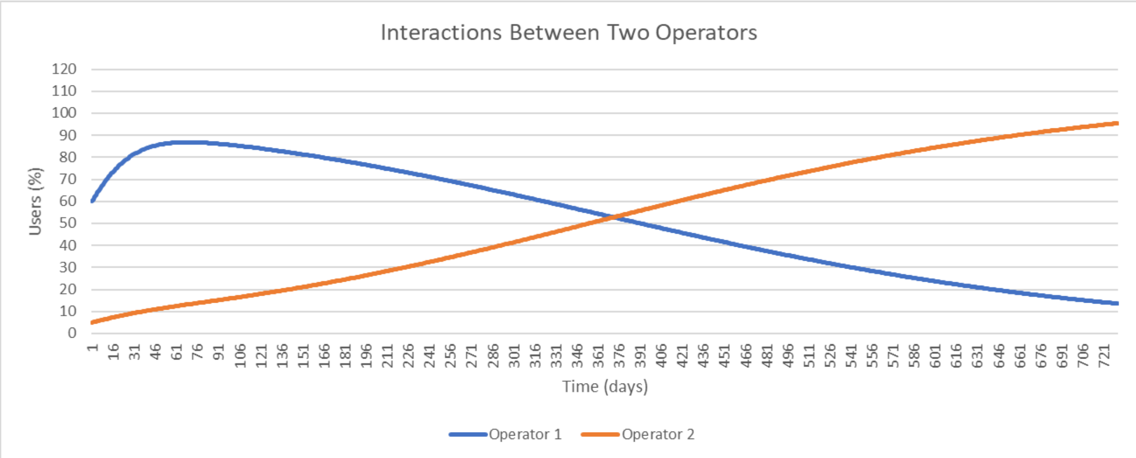

Figure 3-23: Competition with market inversion and annihilation

On that case, initially the population of the incumbent operator growths due to its carrying capacity. If, did not exist the emerging operator the incumbent would reach the maximum, possible, percentage of users. But, the emerging operator's population starts increase, and the incumbent's decrease. Over the time is possible to observe that the emerging operator attracts all the clients. This culminate in a market position inversion and annihilation of the incumbent operator.

The initial number of users of each operator is a determinant factor in this model. If the initial number of users in the two operators is equal, the parameters weight of operator w_{ij}, and the carrying capacity K_i are more relevant, in the interaction, than growth rate r_i. For example, if the carrying capacity is the same in the two operators, the operator which wins is the one who has the greater number of users. But, if the market shares of the operators are different, the one who wins the interaction is that one that had more carrying capacity.
On the other hand, if one of the operators has, initially, much more users than the other then the growth rate has more relevance than the other parameters.
As was previously described, the parameter w_{ij}, represents the weight of the effect of operator i has on operator j. When this parameter is greater than one, one of the actors of the interaction will be oust. The determining factor that dictates which one will be oust, is the initial number of users. The following table shows the parameters of the equations used in each case presented before.

Parameters	Competition with annihilation - Case 1	Competition with annihilation - Case 2	Competition with annihilation - Case 3	Competition without annihilation – Case 1	Competition without annihilation – Case 2	Competition with market inversion and annihilation
N_1	50%	49%	51%	50%	60%	60%
r_1	0.08	0.1	0.1	0.1	0.05	0.1
K_1	100	100	100	100	80	100
$w_{1\|2}$	1.2	1.2	1.2	0.9	0.7	1.2
N_2	50%	51%	49%	50%	5%	5%
r_2	0.1	0.1	0.1	0.1	0.07	0.1
K_2	100	100	100	100	90	100
$w_{2\|1}$	1.2	1.2	1.2	0.5	0.7	0.9

Table 4 - Parameters of interactions

On the following figure, it is possible to observe the impact of the variation of some parameters of the competition model that has been studied on the present section. The carrying capacity of each operator is 1, and the operator 1 has 80% of the market share and operator 2 had 20%, at the start instant of the simulation. It is also important refer that the growth rate varies between 0.1 and 0.7. And the time period considered it was 100 days.

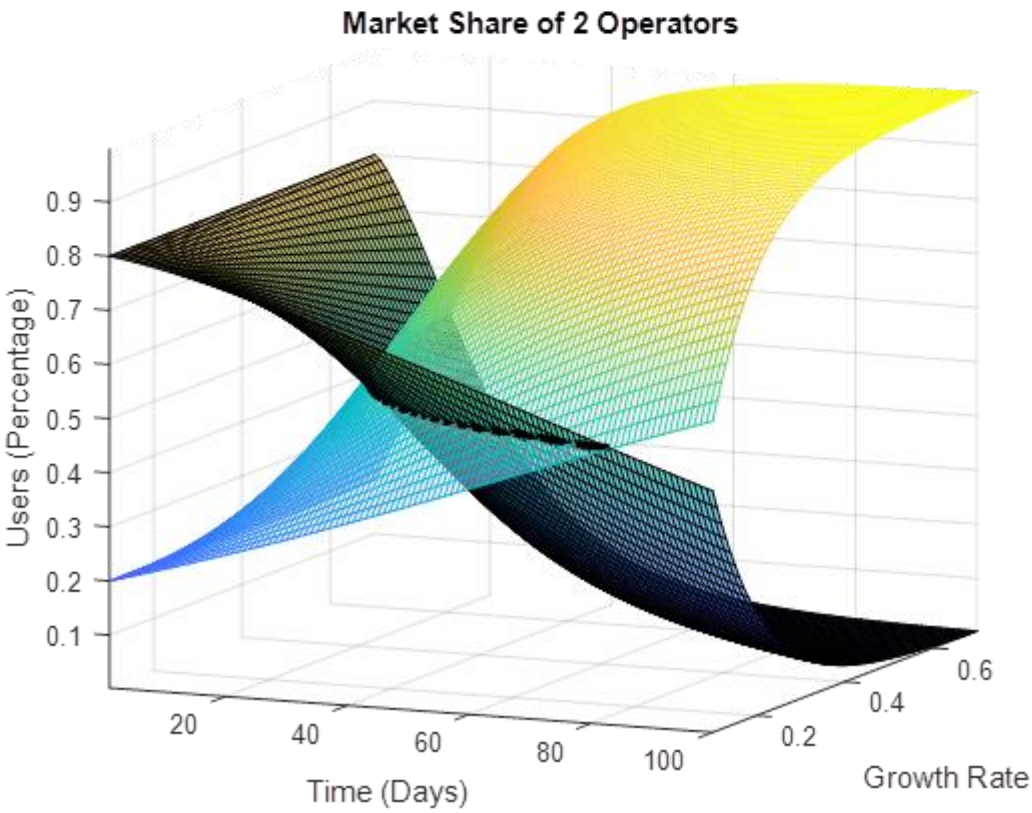

Figure 3-24: Impact of the variation of parameters.

In this case $w_{12} = 1.1$ $w_{21} = 0.8$, which could represent, for example, a case when there is just one operator in market and at some time is launching a new one with same quality of service and best offer in terms of tariff. It is possible see from the above graph that the new operator (colour), reach to the top of the market share, eliminating the incumbent one (black), as faster as bigger the growth rate.

4. Forecasting with historical data

The forecasting based on historical data can be compared to a microscopic vision in the evolution of existing technologies/markets. There is a set of data (time-series), relative to past observations, which are used to predict the future. In this case, it is assumed that some characteristics of the past will occur in the future. The time-series are approached under a deterministic and stochastic way, i.e., every time-series approach can be regarded as the realization of a stochastic process. There are many time-series methods, varying in parameter estimation, identification, model checking and forecasting [20].
Usually, there is available a set of data and the forecast horizon is short, normally days, weeks or months. The most common methods used in this type are moving averages, exponential smoothing and autoregressive models, such as ARIMA (Autoregressive Integrated Moving Average).
The fact is, this type of approach - short term forecasts - is not a handicap but an advantage. In the market, most of the operators, use this type of methodology.

This chapter, will approach some mathematical models, and their main characteristics and fragilities, that are used on data analysis and forecasting. These quantitative models are used to predict future data using the past data. [14]

4.1 Time series

A time series is a sequence of successive observations in time and spaced with constant time intervals. Some examples of a time series are: daily temperature, car sales monthly values, throughput in an interface, hourly CPU capacity usage, etc.

4.1.1 Time series components

In a time series it is possible observe, several patterns. It is helpful be able to classify some of these patterns and behaviours that can be observed. The classical decomposition is a tool that helps to categorize the different elements of a time series. Based on this method each time series can be divided in the following elements:

- **Trend (T) –** long-term increase or decrease in data.
- **Cyclical variation (C) –** Rises and falls that are not a fixed period.
- **Seasonality (S) –** Patterns of change influenced by seasonal factors. Seasonality is of a fix and known period.
- **Irregularity (I) –** What is left over when the other components are extracted from the data, i.e. it is the residue of the data.

Seasonality and cycle can be confused. However, there are different things. While seasonality consists in a fixed and well-known period of changing patterns, the cycles do not consists in fixed periods of time. The period of the cycles, normally, is no longer than the seasonal element.

The classical decomposition is the base for almost all the methods present in this work and can describe almost every type of data. Furthermore, it helps time series and improve forecasts.

4.2 Moving Average

The moving average technique consists in a method to smooth a time series based on the average of the actual observation and the observations in its vicinity. It is usual use this technique to estimate the trend component. [21]

4.2.1 Moving Average Filter

The moving average process can be represented by a finite impulse response (FIR) filter. The simple moving average that will be present on this section is the simplest form of a FIR filter, with all coefficients equals.

As the name implies a moving average filter operates through a series of points of the input data to produce a smooth output version of the data. Let x be the input data and y the output, the moving average filter is written as follows:

$$y(t) = \frac{1}{k}\sum_{j=0}^{k-1} x_{t-j} \qquad 16$$

The following figure shows the block diagram of a filter FIR of order N.

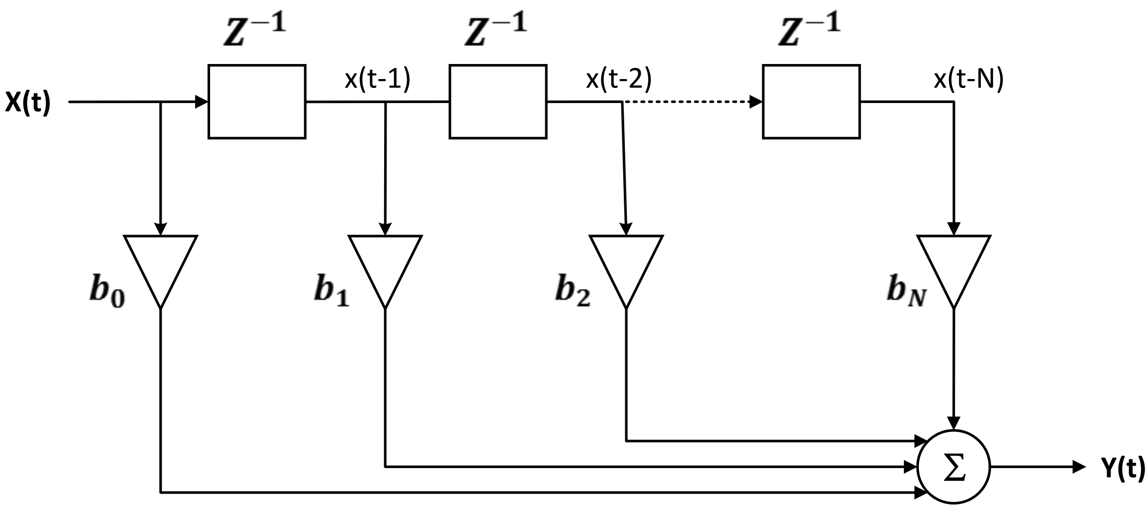

Figure 4-1: Block diagram of a FIR filter of order N [14].

4.3 Exponential smoothing

The exponential smoothing method consists in weighted averages of observations from the past. On this method, the weights run down exponentially as observations become older. [21]

4.3.1 Simple Exponential Smoothing (SES)

SES is an adequate method to predict data without trend or seasonal elements [21]. One of the most common methods to forecast data without these components is the naïve method. In the naïve method, the forecasts are equal to the last observation [21]:

$$\hat{y}_{T+h|T} = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{17}$$

For $h = 1, 2, \ldots$

In the average method, all the observations are considered equal. To achieve a method between these two extremes, it was created the concept of the SES. In this method, the forecasts are the result of the weighted averages where the highest weights are related with the recent observations. [21]

$$\hat{y}_{T+h|T} = \alpha y_T + \alpha(1-\alpha) y_{T-1} + \alpha(1-\alpha)^2 y_{T-2} + \cdots \tag{18}$$

α, represents the smoothing parameter and $0 \leq \alpha \leq 1$. This parameter controls the decrease of the weights.
When $\alpha = 1$, $\hat{y}_{T+1|T} = y_T$, and this method produce the same result of naïve method. [21]

4.3.2 Weight average form

At time $t+1$, the forecast produced is the weighted average between the last observation y_t and the last forecast $\hat{y}_{t|t-1}$. [21]

$$\hat{y}_{t+1|t} = \alpha y_t + (1-\alpha)\hat{y}_{t|t-1} \tag{19}$$

For $t = 1, \ldots, T$, in which $0 \leq \alpha \leq 1$ is the smoothing parameter.

So, let l_0 be the first forecast the first forecast of y_1, then

$$\begin{aligned} \hat{y}_{2|1} &= \alpha y_1 + (1-\alpha) l_0 \\ \hat{y}_{3|2} &= \alpha y_2 + (1-\alpha)[\alpha y_1 + (1-\alpha) l_0] \\ &= \alpha y_2 + \alpha(1-\alpha) y_1 + (1-\alpha)^2 l_0 \end{aligned}$$

The generalized result is given by,

$$\hat{y}_{T+1|T} = \sum_{j=0}^{T-1} \alpha(1-\alpha)^j y_{T-j} + (1-\alpha)^T l_0 \tag{20}$$

It is then possible to conclude that the weighted average form results in the same forecasting equation seen earlier. [21]

4.3.3 Component form

The component form is another way of represent the exponential smoothing. This representation have two equations (forecast and smoothing), for all of the components of the method. For SES the forecast equation is $\hat{y}_{t+1|t} = l_t$, and the smoothing equation is $l_t = \alpha y_t + (1-\alpha)l_{t-1}$. [21]

In which,

$l_t = the\ level\ of\ data\ at\ time\ t$
$\alpha = smoothing\ parameter$
$y_t = observation\ at\ time\ t$

4.3.4 Error correction form

Another way of simple exponential smoothing derives from the equation of level in the form of component arriving at the error correction form $l_t = l_{t-1} + \alpha(y_t - l_{t-1}) = l_{t-1} + \alpha e_t$ [21]

Where

$e_t = y_t - l_{t-1} = y_t - \hat{y}_{t|t-1}\ is\ the\ sample\ forecast\ error\ for\ t = 1, \dots, T.$

4.4 Holt-Winters Exponential Smoothing

When the time series presents seasonality and trend it is common to use the Holt-Winters model. This model is an improvement of SES. [21]
To produce forecasts, this model make use of two smoothing equations [21]:

For level, is calculated a weighted average between the observation y_t and estimated level based on $l_{t-1} + b_{t-1}$:

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + b_{t-1})$$

For trend, is calculated a weighted average of the estimations of trend at time t $(l_t - l_{t-1})$ and b_{t-1}:

$$b_t = \beta^*(l_{t-1} + l_{t-1}) + (1-\beta^*)b_{t-1}$$

With the result of these two equations the forecasts are obtained with:

$$\hat{y}_{t+h|t} = l_t + hb_t$$

That is the estimation of level plus h multiplied by the estimation of trend. The parameters presented in the above equations are:

$l_t = estimated\ level\ of\ the\ series\ at\ time\ t$
$b_t = estimated\ trend\ of\ the\ series\ at\ time\ t$
$h = forecast\ horizon$
$\alpha = smoothing\ parameter\ for\ level$
$\beta^* = smoothing\ parameter\ for\ trend$

$y_t = observation\ at\ time\ t$

The parameters α and β^*, are both between 0 and 1. [21]

4.4.1 Damped trend methods

The result of Holt-Winters linear method produces predictions whose trend is constantly increasing or decreasing. [21] It is empirically known that this method produces over-forecasts, especially when forecasting horizons are long. [21]
To overcome this limitation of the linear method of Holt-Winters, the damped trend methods appeared.

4.4.1.1 Additive damped trend

The main difference between Holt-Winters linear method and the additive damped trend, is the damping parameter $0 < \phi < 1$ [21]:

$$l_t = \alpha y_t + (1-\alpha)(l_{t-1} + \phi b_{t-1})$$
$$b_t = \beta^*(l_t - l_{t-1}) + (1-\beta^*)\phi b_{t-1}$$
$$\hat{y}_{t+h|t} = l_t + (\phi + \phi^2 + \cdots + \phi^h) b_t$$

If $\phi = 1$ the result is the Holt-Winter linear method. If ϕ is between 0 and 1, the trend is damped. [21] For any value $0 < \phi < 1$, the forecasts converge to $l_T + \frac{\phi b_T}{(1-\phi)}$ as $h \to \infty$. This results in trended short-term forecasts and constant long-term forecasts. [21]

The error correction form of the smoothing equations is $l_t = l_{t-1} + \phi b_{t-1} + \alpha e_t$ and $b_t = \phi b_{t-1} + \alpha\beta^* e_t$ [21]

4.4.1.2 Multiplicative damped trend

To improve the linear, and additive damped trend methods it was introduced the multiplicative damped trend method [21]:

$$l_t = \alpha y_t + (1-\alpha) l_{t-1} b_{t-1}^{\phi}$$
$$b_t = \beta^* \frac{l_t}{l_{t-1}} + (1-\beta^*) b_{t-1}^{\phi}$$
$$\hat{y}_{t+h|t} = l_t b_t^{(\phi + \phi^2 + \cdots + \phi^h)}$$

With this method it is possible produce more flexible predictions than with the other two methods [21].

The error correction form of the smoothing equations is $l_t = l_{t-1} b_{t-1}^{\phi} + \alpha e_t$ and $b_t = b_{t-1}^{\phi} + \alpha\beta^* \frac{e_t}{l_{t-1}}$. [21]

4.5 Auto-Regressive Moving Average

The Box & Jenkins (1970) methodology is widely used in time series analysis. This methodology consists in the adjustment of Auto-Regressive (AR) Integrated Moving Average models – ARIMA (p, d, q) – to a set of data.

The ARIMA models, derive from mathematics and statistics tradition and can be applied to a great number of very different phenomena. Based on the literature one might expect that the ARIMA models to work well, as predictors, in the case of relatively stabilized phenomena, where there is a strong stochastic component, for which many data points are available (not less than 30, according to the basic work of Box and Jenkins, 1970). [10]

In time series analysis, the auto-regressive moving average model (ARMA) is often used, and three different methodologies can be used: AR (p), MA (q) and the combination of the two that holds ARMA (p, q).

When differentiation is performed, the combination AR (p) and MA (q) is called ARIMA (p, d, q), the letter I refers to the differentiation procedure.

Before introducing the ARMA model, its important introduce the concept of stationarity.

Stationarity - A stationary time series is one whose properties do not depend on the time at which the series is observed. [21]

Properties such as mean, autocorrelation, etc. should be constant over the time to the series be considered stationary. Trend, seasonality and their combination are examples of non-stationary behaviours.

The ARIMA model can deal with non-stationary time series since it does not exhibit an explosive behaviour.

The application of an ARIMA model implies that the time series is stationary, otherwise the predicted values may result in poor prediction.

A non-stationary time series can be transformed into stationary by differentiating consecutive observations ($y_t' = y_t - y_{t-1}$).

There are different forms and types of differentiation, such as: first-order differentiation, seasonal differentiation, etc.

4.5.1 Auto-Regressive model

The AR model is a subset of ARMA models. It's called Auto-Regressive (AR) because the output values are calculated based on the regression of previous values, it behaves as a multiple regression where predictors are delayed values of y_t. The AR processes can be represented by a simple infinite response filter block diagram (IIR).

In the AR model, the output values are calculated based on the regression of the previous output values and their expression can be presented as follows:

$$\mathrm{y_t} = \sum_{\mathrm{k=1}}^{\mathrm{p}} \phi_k \mathrm{y_{t-k}} + \mathrm{e_t} \qquad 21$$

where:

- ϕ_k auto-regressive coefficient;
- p order of the filter (or the process AR);
- e_t residuals.

The same can be represented by a block diagram of an Infinite Response Filter (IIR) filter as shown in the following figure:

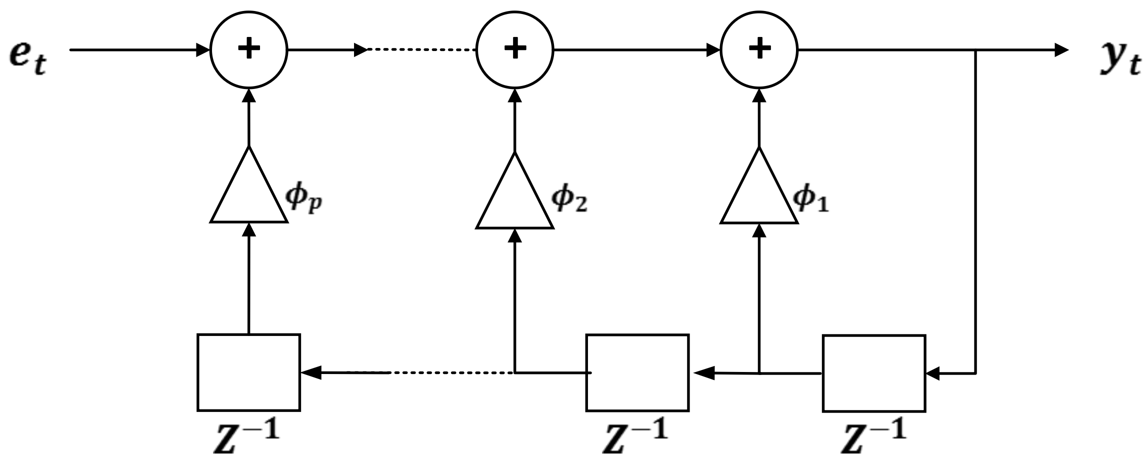

Figure 4-2: p-order IIR filter implementing an auto-regressive model [14]

Stationarity

It is assumed in many time series techniques that the data is stationary. A stationary process has the property that the mean structure, variance, and autocorrelation does not change over time. A stationary series can be defined as a series of flat appearance with no trend, constant variance over time, an autocorrelation structure constant over time and without periodic fluctuations (seasonality).

4.5.2 Moving Average model

Another subset off the ARMA model is the Moving Average (MA). The moving average model uses past forecast errors in a model like regression.

Thus, a moving average model of order q – MA(q) – can be denoted as:

$$y_t = \sum_{k=1}^{q} \theta_k e_{t-k} + e_t \qquad 22$$

Where e_{t-q} are the residuals in the lag q and $\theta_k$are the moving average coefficients. This moving average coefficients (θ_k) must respect the stationarity constraints $-1 < \theta_k < 1$.

The moving average model is a FIR filter, but instead of using past observations (x_t) as input, uses the residuals e_t. The following figure shows a block diagram representation of a residuals moving average filter.

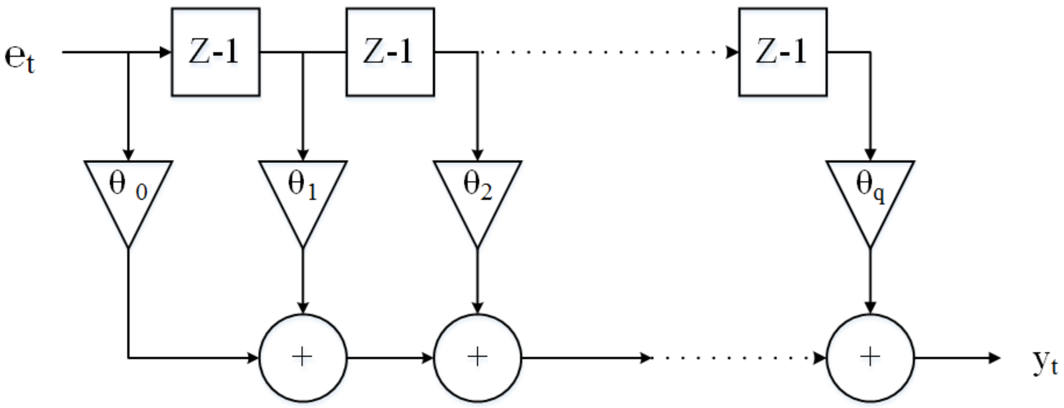

Figure 4-3: q-order filter implementing a moving average model [14]

4.5.3 Auto-Regressive Moving Average model – ARMA model

From the combination of the Auto-Regressive (AR) model and the Moving Average (MA) model, presented above, it results the Auto-Regressive Moving Average model (ARMA). The mathematical equation that represents the model is:

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \cdots + \theta_q e_{t-q} + e_t \qquad 23$$

This model uses the delayed values of y_t as well as the delayed values of the residuals e_t. In ARMA notation (p, q), the letter p refers to the order Auto-Regressive model and q to the order of Moving Average model. The ARMA model can be represented by the combination of a FIR (with the input equal to the residues) and a IIR filters, in fact, the representation of the ARMA filter is the combination of the AR and MA filters discussed above.

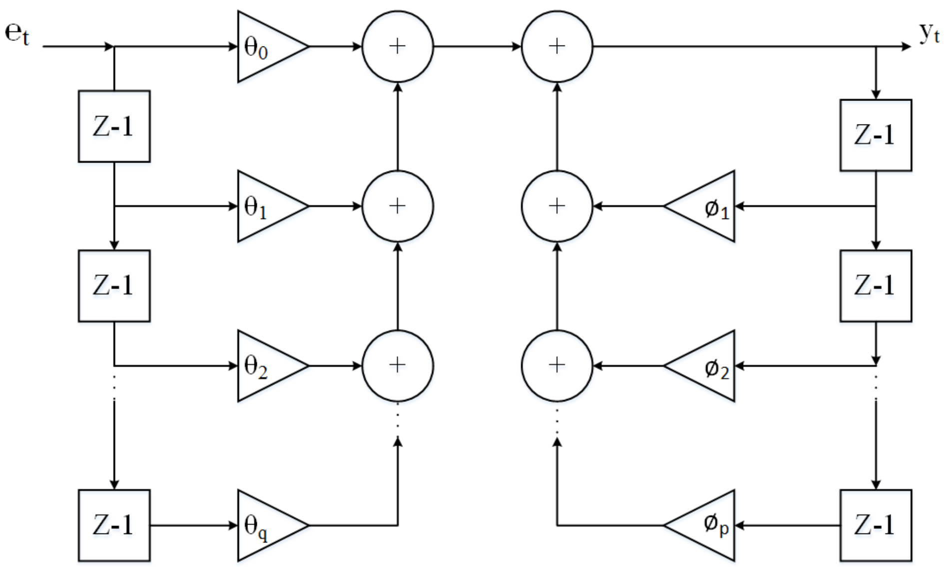

Figure 4-4: FIR/IIR filter implementing a Non-Seasonal ARMA(p,q) model [14]

4.6 Forecast evaluation

A very important step in the use of a quantitative prediction method is to understand the performance of the method in this type of data.

This section presents some of the most common measures used to evaluate the accuracy of forecasting methods and the quality of fit.

4.6.1 Sum of Squared Errors

The forecast error is simply the subtraction between the real value and the forecast value, i.e. $e_t = y_t - \hat{y}_t$.

Accuracy measurements based on e_t has the property of being scale-dependent and cannot be used to make comparisons between different scales.

A common measure based on the forecast errors is the Sum of Squared Errors (SSE) [22]:

$$\mathrm{SSE} = \sum_{t=1}^{n} e_t^2 \qquad 24$$

4.6.2 Mean Absolute Percentage Error

The Mean Absolute Percentage Error (MAPE) [22] is the most commonly used measure to compare performance between different data sets. In a simple way, the MAPE is the average of the relative errors (in percentage). Thus, let y_t be the actual values and $\hat{y}_t$ the forecasted values, then:

$$\mathrm{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left(\left|\frac{y_t - \hat{y}_t}{y_t}\right| \times 100\right) \qquad 25$$

n → Number of observations

The forecasting method that minimizes the MAPE is considered the preferred method for forecasting.

The MAPE concept although being scale-independent and very simple, it has two main drawbacks in practical applications:

- If there are zero values, there will be a division by zero.
- A null MAPE means a perfect fit of the model. But there is no upper limit for MAPE values.

4.6.3 Mean Square Error

The mean squared error shows how close a regression line is to a set of data. This is possible taking the distances from the points to the regression line (these distances are the “errors”) and squaring them [23].

$$\mathrm{MSE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}} \mathrm{e}_t^2 \qquad 26$$

4.6.4 Mean Absolute Error

Measures the average magnitude of the errors in a series of predictions, without considering their direction. It is the mean over the sample of the absolute difference between the prediction and the observation. [24]

$$\mathrm{MAE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{t}=1}^{\mathrm{n}} (|\mathrm{y_t} - \hat{\mathrm{y}}_\mathrm{t}|) \qquad 27$$

4.7 Model Selection

The estimation of the model is the identification of the order of the model, for a given set of data. This identification is done "manually" using the Autocorrelation Function (ACF) and the Partial Autocorrelation Function (PACF).

4.7.1 Autocorrelation function and Partial Autocorrelation function

Given the observations $Y_1, Y_2, Y_3, \dots, Y_N$ at time $t_1, t_2, t_3, \dots, t_N$, the lag h autocorrelation function is defined as

$$r_h = \frac{\sum_{i=1}^{N-h}(Y_i - \hat{Y})(Y_{i+h} - \hat{Y})}{\sum_{i=1}^{N-h}(Y_i - \hat{Y})^2} \qquad 28$$

For the autocorrelation, the assumption is that the observations are equi-spaced. [25]

Partial autocorrelation is a commonly used tool for model identification. The partial autocorrelation at the lag h is the autocorrelation between Y_t and , Y_{t-h} that is not accounted for by lags 1 through $k-1$.

Specially, partial autocorrelations are useful in identifying the order of an autoregressive model. The partial autocorrelation of an $AR(p)$ process is zero at lag $p+1$ and greater. If the sample autocorrelation plot indicates that an AR model may be appropriate, then the sample partial autocorrelation plot is examined to help identify the order. We look for the point on the plot where the partial autocorrelations essential becomes zero. Placing a 95% confidence interval for statistical significance is helpful for this purpose. [25] The approximate 95% interval for partial autocorrelations are at $\pm\frac{2}{\sqrt{N}}$.

The observation of ACF and PACF to identify ARMA models is something more subjective than objective. What makes this model identification technique a difficult task for the analysts of the time series.

To resolve this problem, time series analysts try to use alternative objective methods for identifying ARMA models. One of these criteria, and most used, is the Akaike Information Criterion (AIC).

4.7.2 Akaike Information Criterion

The AIC criterion is a measure of relative quality of a model. This criterion does not tell nothing about the quality of the model in absolute sense, i.e. if the models to be compared fit poorly the data, AIC does not inform about that. AIC calculates a trade-off between the goodness of fit and complexity of the model, thereby providing a means for model selection. In equation form:

$$\mathrm{AIC} = \ln(\hat{\sigma}_{\varepsilon}^{2}) + \frac{2\mathrm{k}}{\mathrm{T}} \qquad (29)$$

$\mathrm{k} \rightarrow$ Number of the estimated ARMA parameters $(p + q)$
$\mathrm{T} \rightarrow$ Number of observations
$\hat{\sigma}_{\varepsilon}^{2} \rightarrow$ Variance of the residuals

The model who generates an AIC with the lower values is the model who best fit the time series. However, it does not mean that this model produces the best forecast.

An alternative, or additional, criterion to AIC and widely used is also the minimization of Sum of Squared Errors (SSE). The difference is that while the AIC assumes a trade-off between the complexity / order of the model and the minimization of errors, the minimization of SSE only considers the minimization of errors.

The figure below gives an overview of the major steps of the time series analysis's:

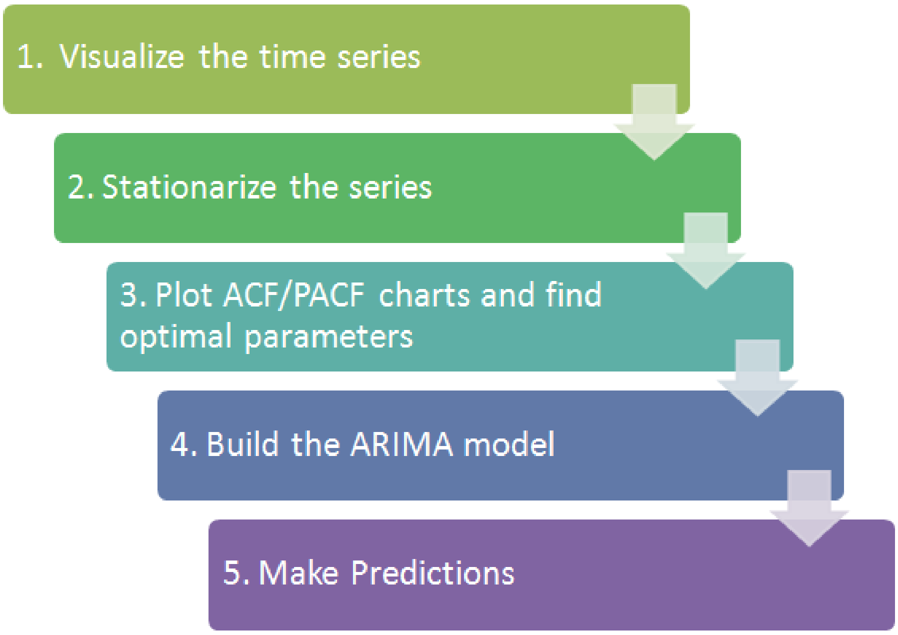

Figure 4-5: Time Series Analysis [26]

5. Combining different methodologies of forecasting

On this section, it was tried to combine different forecasting methodologies in a telecommunications network scenario.

5.1 Case study of combination of Forecast Methodologies

Let's consider a case when a technology, for example UMTS, is in the market and at certain point a new technology, for example LTE, is launched. LTE, is a new generation of technology and could be considered an improvement of UMTS capable of increasing the intensity of usage of already existent users and also attracting new users.

In this case study, it is attempted to combine the two types of forecasting studied before. The long-term forecasting, i.e., the prediction based on logistic models which describe the penetration of some technology or product in market, and the short-term forecasting based on historical data. As will become clear at a later stage, the end result is the ability of producing short term predictions based not only on historical data but also on changes derived from the dynamics associated with the introduction of new technologies (for which historical data is not available).

The graph below, represents ten years of monthly data. These data describe the use of a cellular network element, for example, the load of an RNC at a specific time of a specific day of a series of months.

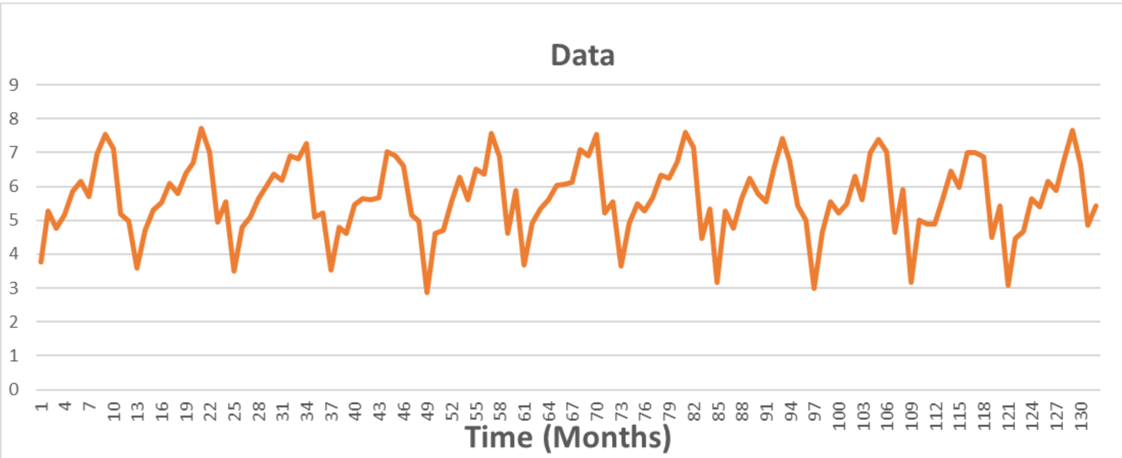

Figure 5-1: Data series of use of a cellular network

To model It has been assumed that he penetration of UMTS and LTE in the market follow logistic laws: the two curves represented in Figure 5-2.

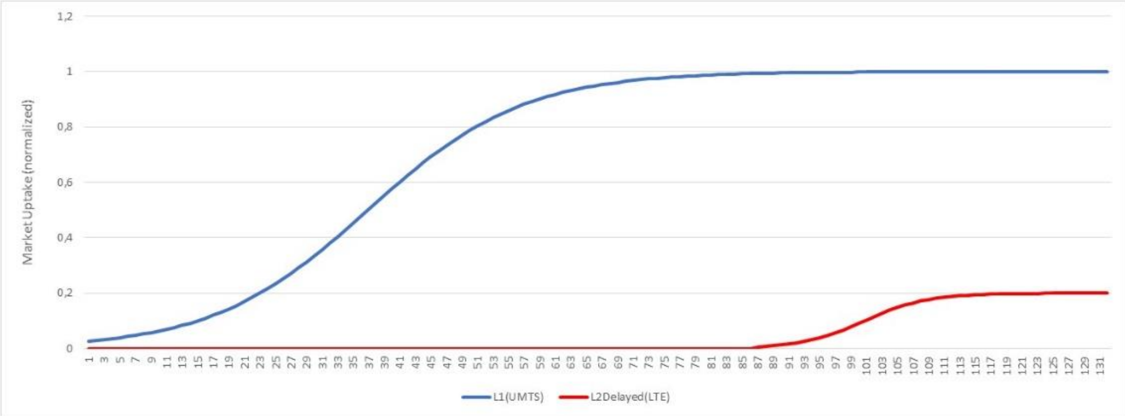

Figure 5-2: Logistic curves UMTS (Blue) LTE (Red)

The general expression for these curves has been introduced in section 3 and is the following:

$$P(t) = P_i + (P_f - P_i)\frac{1}{1 + \alpha e^{-\beta t}} \qquad 30$$

In figure 5-2 the following logistic curve parameters have been assumed:

	UMTS	LTE
α	40	10
β	-0.1	-0.18
P_i	0	0
P_f	1	0.2

Table 5: Parameters of Logistic Model of Figure 5-2.

The model which describes the penetration of LTE, it was delayed 86 months (approximated 7 years), because this technology was launched after UMTS be completely established in the market.

Then the historical data presented before it was modulated with this to models, and the results are showed in the next two figures:

UMTS:

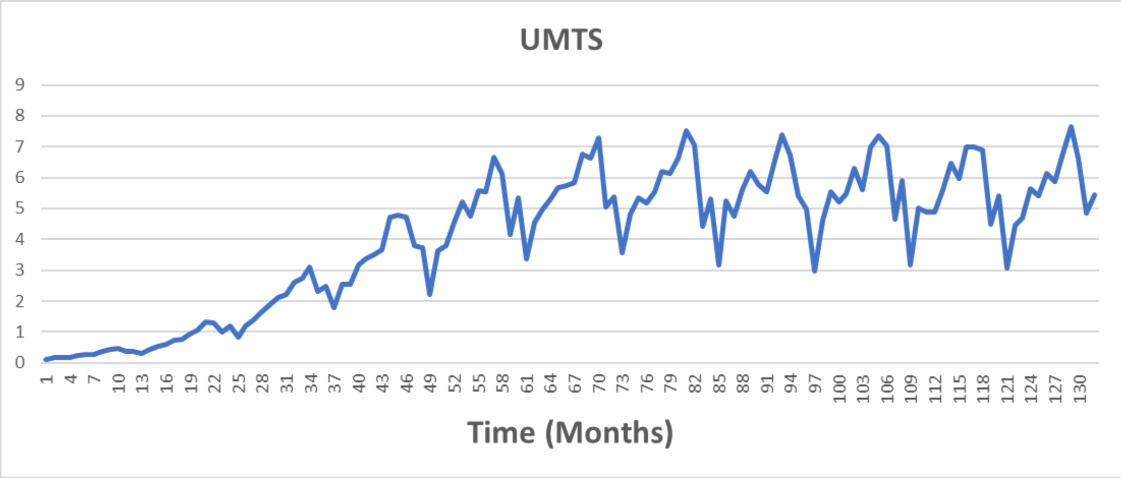

Figure 5-3: Time series that describes the usage of UMTS network

LTE:

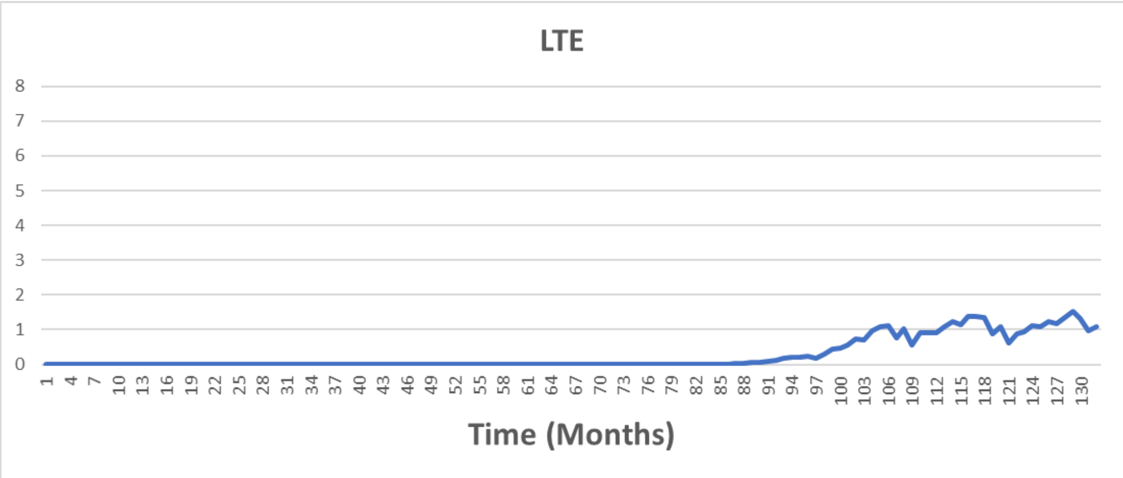

Figure 5-4: Time series that describes the usage of LTE network

And finally, the two models were summed, to obtain a time series which describes the penetration of the two technologies, in the market over 11 years, and the utilization patterns of the network monthly. The result is the following:

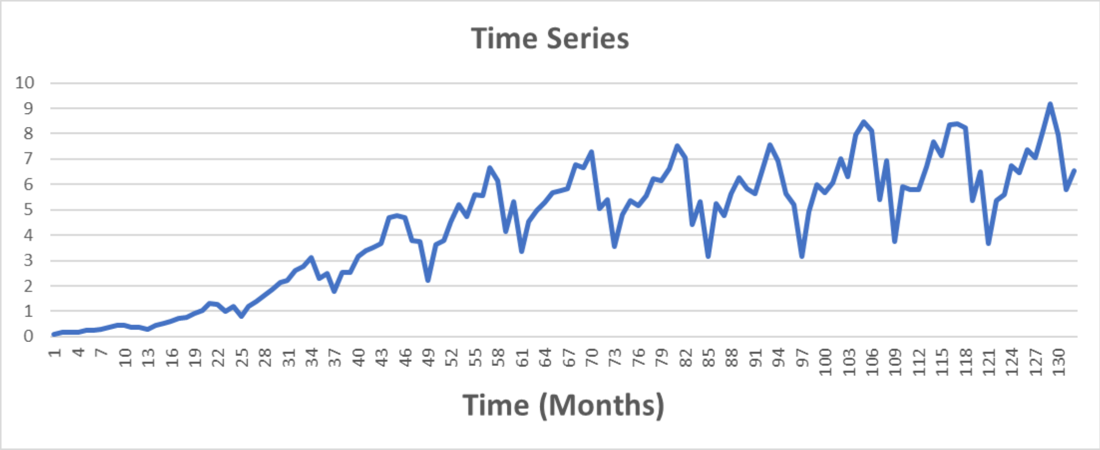

Figure 5-5: Time series that describes the usage of network

It is possible to observe that the entry of LTE in the market confers a 20% increase in the number of users and slightly increases the usage patterns already observed in UMTS. Some randomness was also added, so that the data series would look more realistic.

5.2 Forecasting approaches and results

Two approaches have been used in this study to forecast the previous time series. The first is the simple application of several methods to the time series and compare which one the has the best accuracy prediction results. The second consists in a decomposition of the time series apply an adequate method to forecast each method and compare the accuracy prediction results with the ones obtain with the first approach.

5.2.1 First approach

The first approach consists in the use of the functions of several models of the package *Forecast* developed by Rob J. Hyndman for R [25] to forecast the time series and observe which one produce the best fit, based on accuracy tests.

The models used were the Simple Exponential Smoothing, Holt-Winters Multiplicative, Holt-Winters Additive and Arima. For these models were performed accuracy tests and it will be present the graphical results of the predictions and the values of the MSE and MAPE. The forecast horizon it was 12 months (1 year), in all methods.

Time series:

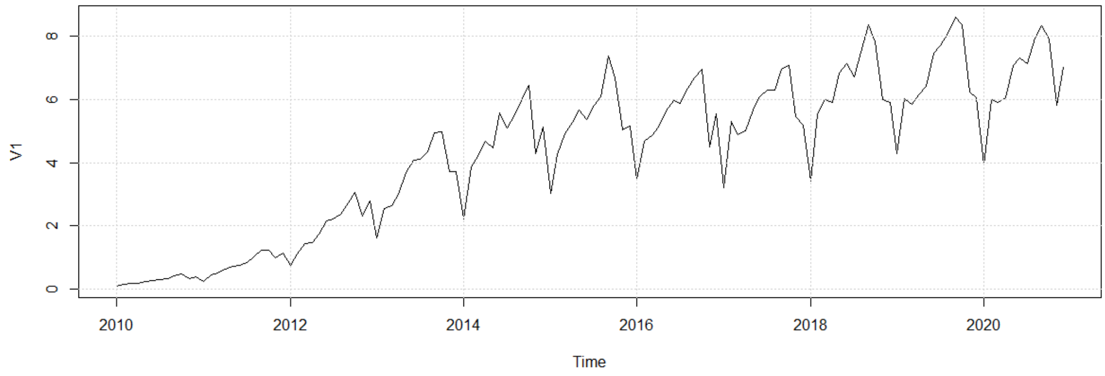

Figure 5-6: Time series

This is the time series presented before. It was used to test the methods mentioned before.

To identify the order of the auto regressive model it was used the ACF (Auto-Correlation Function) and PACF. The graphical aspect of the ACF, of the used time series, is the following:

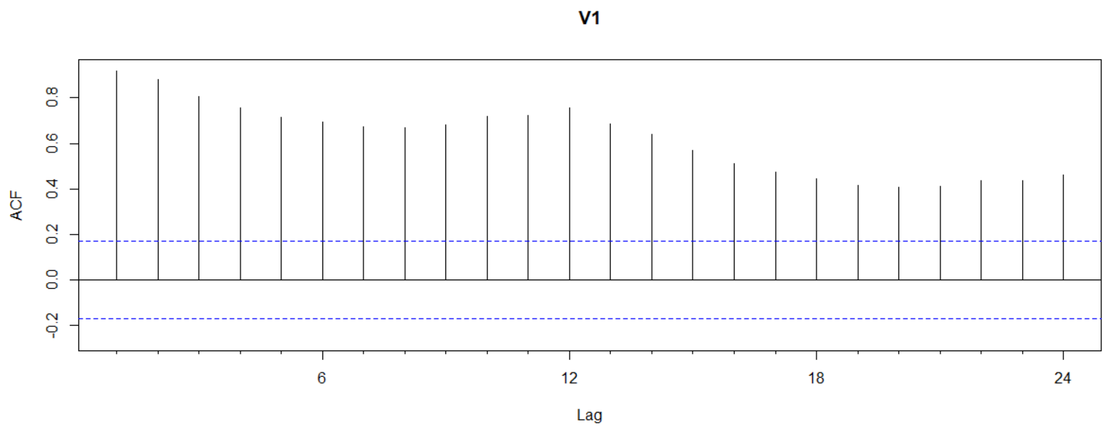

Figure 5-7: ACF

And the graphical aspect of the PACF:

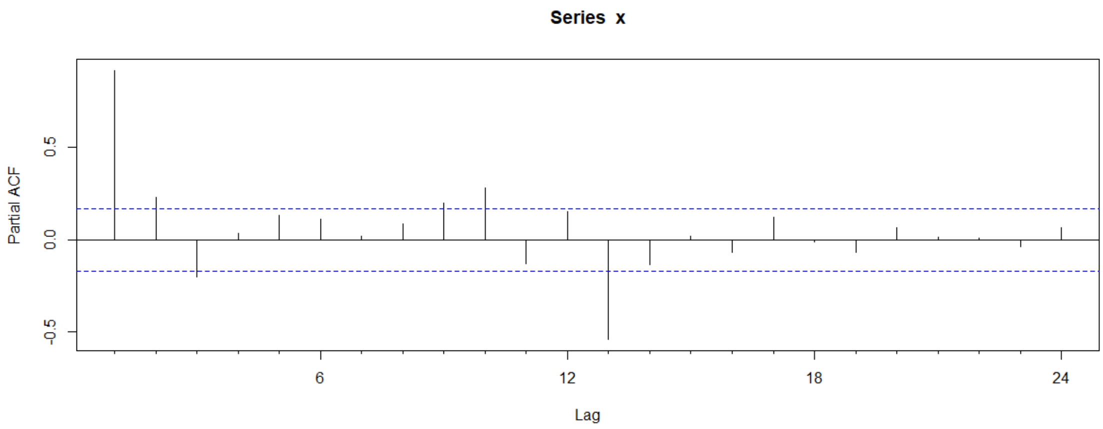

Figure 5-8: PACF

By the observation of these graphical results it was identified the ARIMA model presented in the end of this subsection.

Simple Exponential Smoothing:

This is the result of the SES (Simple Exponential Smoothing) method with a forecast horizon of 12 months.

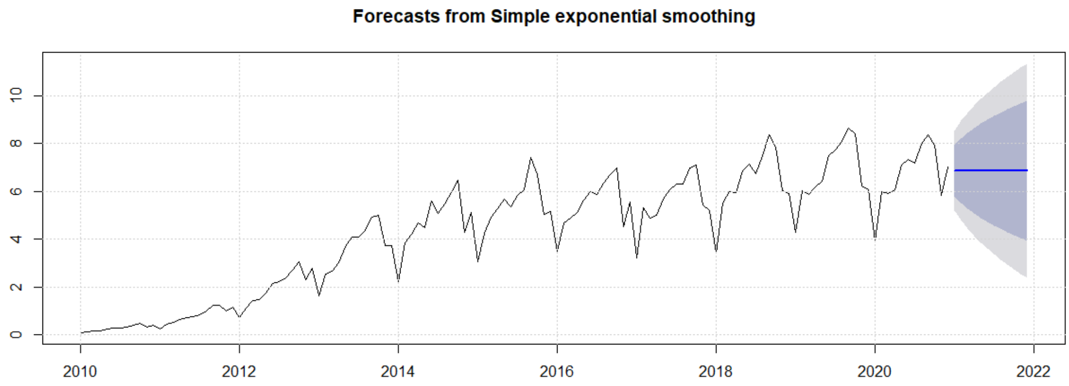

Figure 5-9: Plot of forecast with SES

It is possible observe that the prediction, (blue line on figure), is not enough accurate and the confidence interval is bigger, (grey area on figure). This result it was the expected because SES consists in weighted moving averages of the past observations, and it is not an adequate method to apply in time-series with trend and seasonality.

The results of the accuracy test were:

$Mean\ of\ Time\ Series\ (Mean)$	4.36
MSE	0.71
$MAPE$	17.48%
$\frac{MSE}{Mean^2}$	0.037

Table 6: Accuracy results for SES

Holt-Winters Multiplicative:

This is the result of the Holt-Winters Multiplicative method whit a forecast horizon of 12 months.

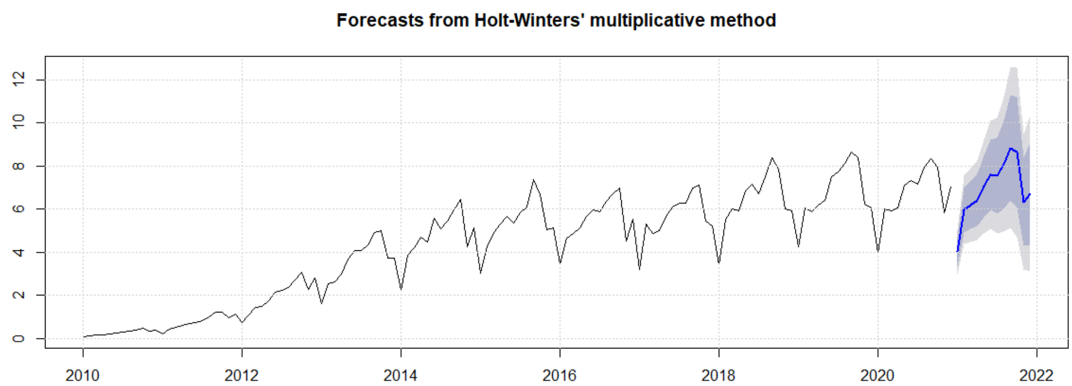

Figure 5-10: Plot of forecast with HW - Multiplicative Method

On that case, the prediction seems accurate, (blue line on figure), but the confidence interval it is bigger (grey area on graph). However, it is an adequate method to data series like the one used in this studied, because seasonal variations are changing proportional to the level of the series.

The results of the accuracy test were:

$Mean\ of\ Time\ Series\ (Mean)$	4.36
MSE	0.065
$MAPE$	6.27%
$\frac{MSE}{Mean^2}$	0.0034

Table 7: Accuracy results for HW - Multiplicative

Holt-Winters Additive:

This is the result of the Holt-Winters Additive method whit a forecast horizon of 12 months.

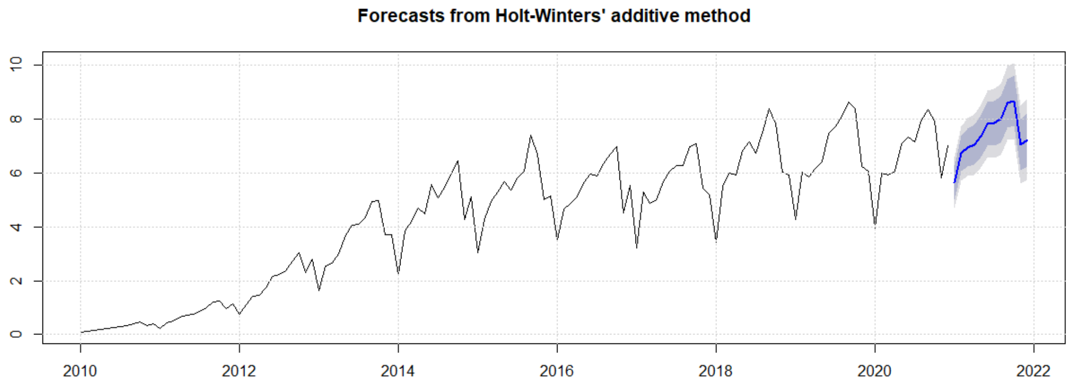

Figure 5-11: Plot of forecast with HW - Additive Method

On that case, the prediction seems less accurate, (blue line on figure), and the confidence interval it is shorter (grey area on graph). As it was studied in section 4.4.1.1, this method produces forecasts more conservatives than the HW – Multiplicative which is reflected in the accuracy results.

The results of the accuracy test were:

$Mean\ of\ Time\ Series\ (Mean)$	4.36
MSE	0.23
$MAPE$	41.48%
$\frac{MSE}{Mean^2}$	0.0121

Table 8: Accuracy results for HW - Additive

ARIMA:

This is the result of the ARIMA method whit a forecast horizon of 12 months.

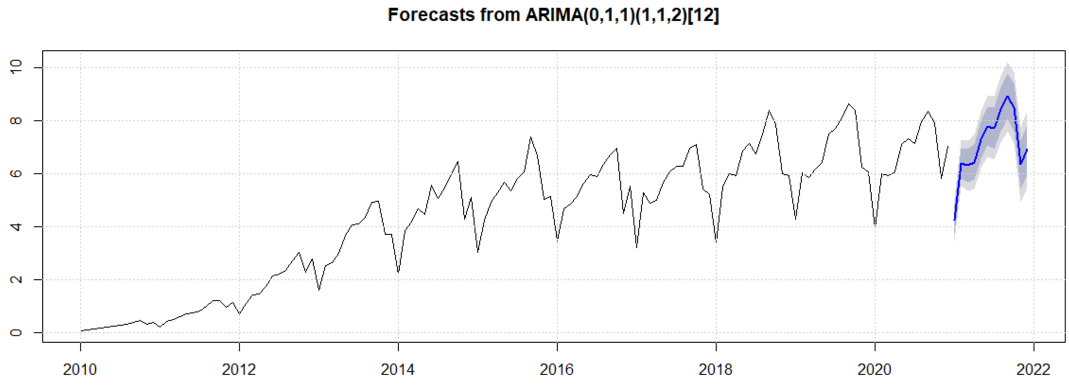

Figure 5-12: Plot of forecast with ARIMA

It is possible observe that the prediction, (blue line on figure), is accurate and the confidence interval is smaller, (grey area on figure). However, the accuracy results demonstrate that the ARIMA model it is, on that case, less accurate than HW – Multiplicative one.

The results of the accuracy test were:

$Mean\ of\ Time\ Series\ (Mean)$	4.36
MSE	0.14
$MAPE$	7.51%
$\frac{MSE}{Mean^2}$	0.0074

Table 9: Accuracy results for ARIMA

5.2.1.1 Results discussion

Looking to the MAPEs values is possible observe that the HW (Holt - Winters) Multiplicative, followed by the ARIMA, are the models that best fit the forecast. The HW Additive is the one who had the worst result in terms of MAPE followed by SES.

In terms of MSE, the best method is the HW Multiplicative, ARIMA, HW Additive, followed by SES. The MSE is the sum of the square of errors divided by the number of the observations. And the errors are the difference between the observation and the predict value. So, to have a realistic perspective of the magnitude the accuracy of the method, it has been divided the values of the MSE of the for methods by the square of the mean of the time series. Taking in account this fact, again, the HW – Multiplicative is the most accurate method followed by the ARIMA, HW – Additive and SES.

It was divided the MSE by the square of the mean to test the accuracy of the studied models. It is not a common approach but it can make sense to take into account the range of values in the time series. Close MSE values in time series with very disparate ranges of values may not have the same meaning. This is a normalization effort. And again, the HW – Multiplicative is the most accurate method.

5.2.2 Second approach

The second approach, consists in the decomposition of the time series in three components. The trend, the seasonality and the random. Then were predicted the trend and the seasonality separately and in the final, all the components were summed again to perform the final time series forecast. To predict the trend, it was used the Simple Exponential Smoothing Method and to predict the seasonality it was used the Holt-Winters Method. The forecast horizon it was 12 months (1 year), for the two components.

The following figure is the block diagram representation of the approach described before:

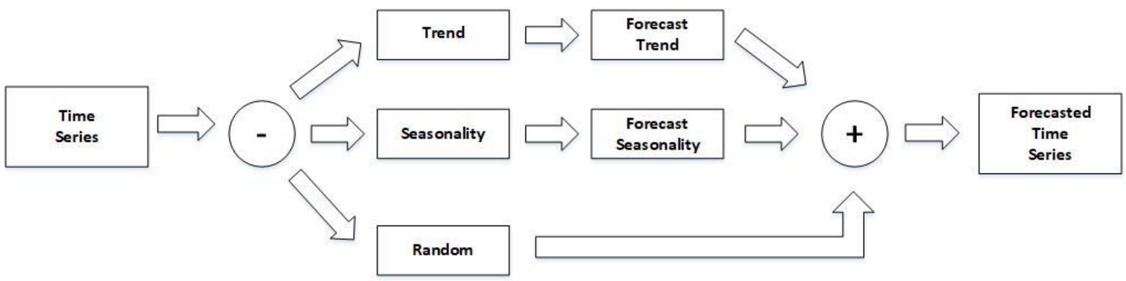

Figure 5-13: Block Diagram of the approach

Time series:

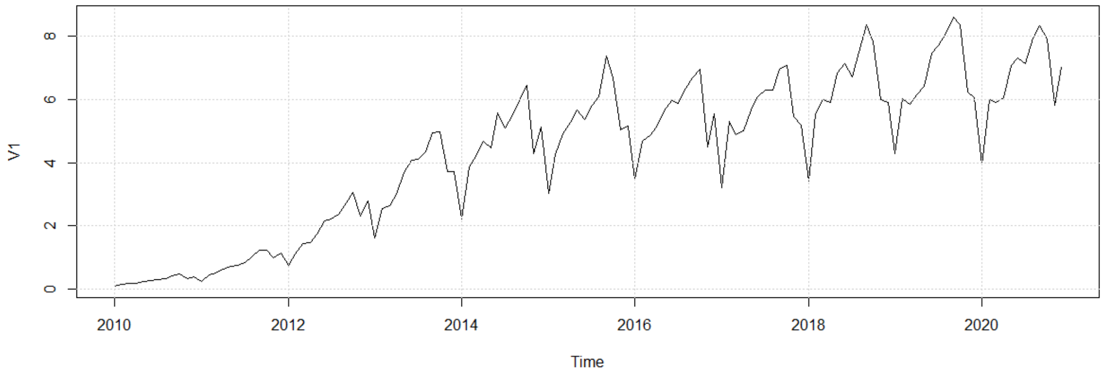

Figure 5-14: Time series

Trend component:

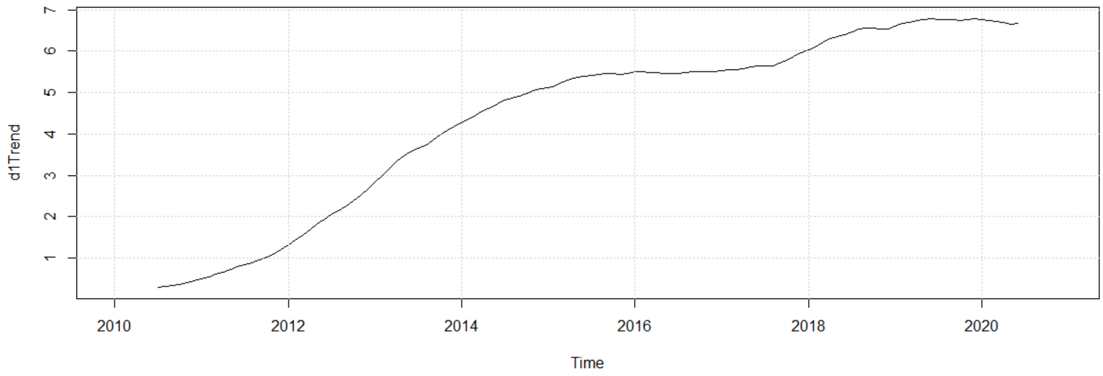

Figure 5-15: Trend component

Trend forecasted:

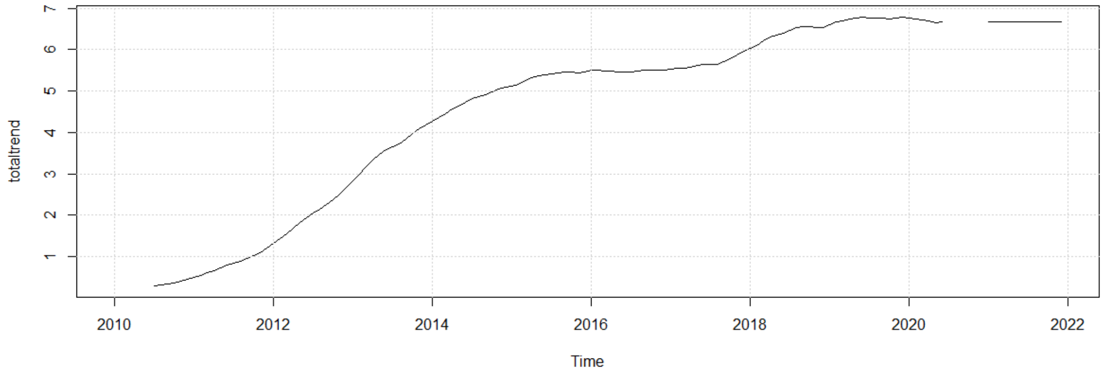

Figure 5-16: Forecasted trend component

The trend it was forecasted using the simple exponential smoothing function and with a forecast horizon of 12 months (1 year).

The results of the accuracy tests of the trend's prediction were the following:

$Mean\ of\ Time\ Series\ (Mean)$	3.89
MSE	0.0053
$MAPE$	2.62%
$\frac{MSE}{Mean^2}$	0.00035

Table 10: Accuracy results for SES used to forecast trend component

Seasonal component:

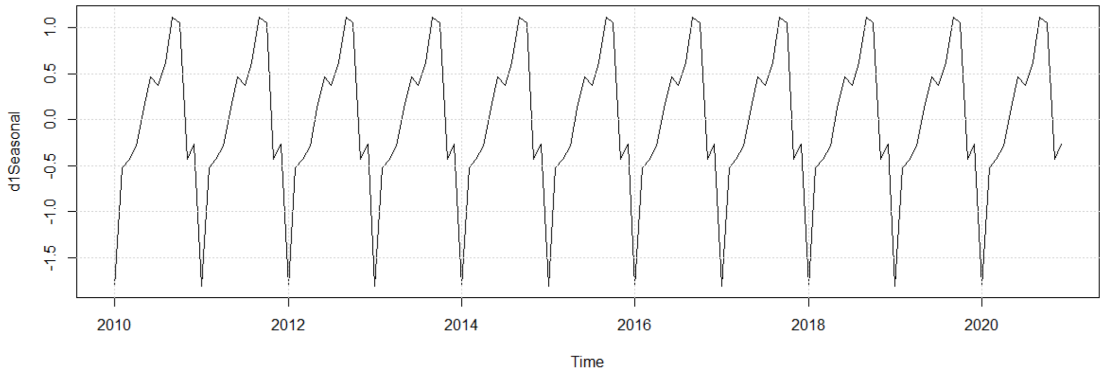

Figure 5-17: Seasonal component

Seasonality forecasted:

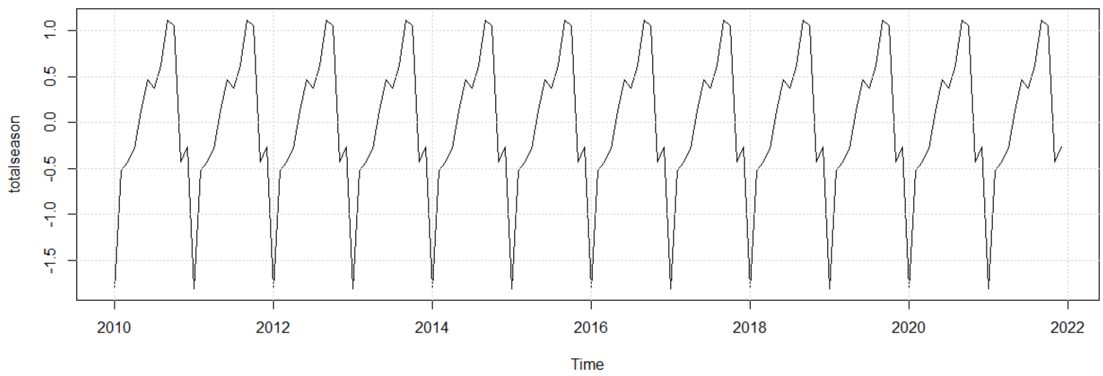

Figure 5-18: Forecasted seasonal component

The trend it was forecasted using the Holt – Winters function and with a forecast horizon of 12 months (1 year).

The results of the accuracy tests of the seasonality's prediction were the following:

$Mean\ of\ Time\ Series\ (Mean)$	3.89
MSE	4.14×10^{-34}
$MAPE$	$3.97\times10^{-15}\%$
$\frac{MSE}{Mean^2}$	2.73×10^{-35}

Table 11: Accuracy results for HW used to forecast seasonal component

Random component:

The random component is not predictable. So it was replicated the same pattern of randomness of the last year, in order to achieve the final prediction of the time series.

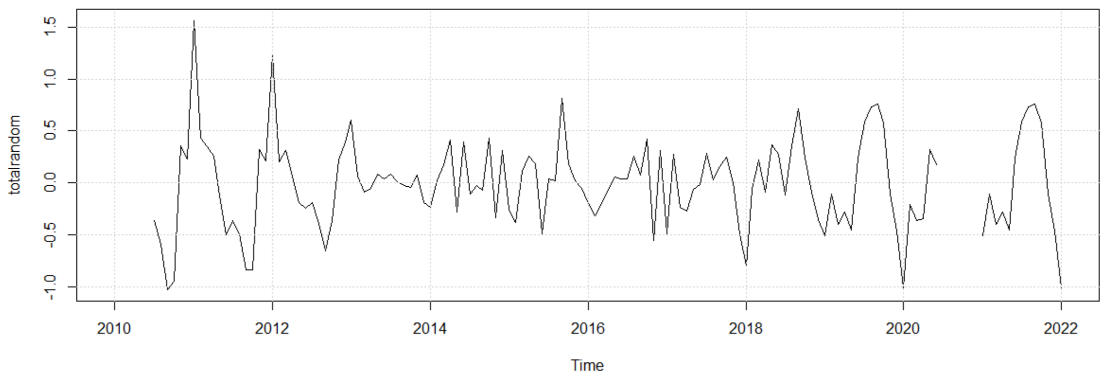

Figure 5-19: Random component

Result of forecast:

The result it is the following (red line in graph):

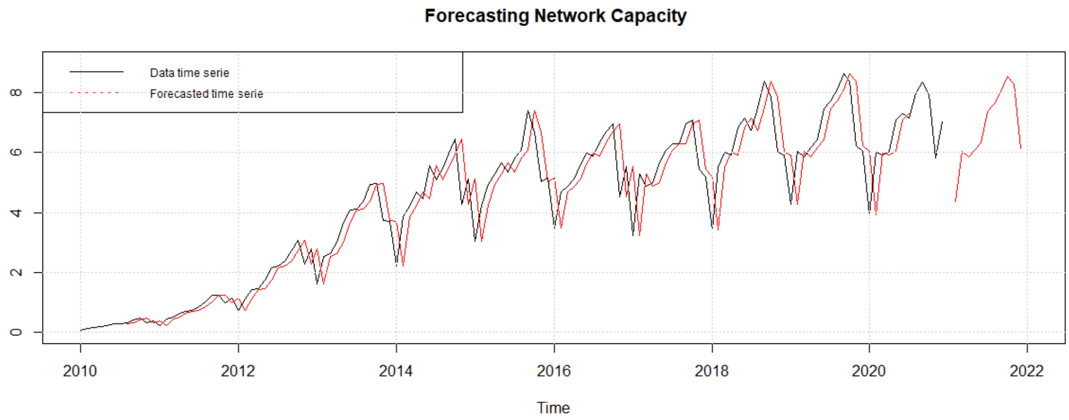

Figure 5-20: Forecasted time series

The black line represents the initial time series. The red line the forecasted time series. It is possible observe the prediction of the next year in red, too.

The result of the accuracy tests of the different methods used to predict the two components, was smaller than the ones obtained to the same methods when applied to the non-decompose time series (First approach).

With these facts is possible that the decomposition approach (Second approach), it is more accurate than the simple application of a certain method to the time series. This means that, the predictions are more accurate when the right method of forecast is applied to the right component of the time series.

Since in the cellular network planning, it is often necessary to deal with update/replacement of technology, which leads to changes and fluctuations in usage/consumption patterns by customers, i.e., changes in long-term trends an approach of decomposition forecasting and forecasting of the various components separately will be the most appropriate in this area of work.

6. Techno-Economic Analysis

In this section, the forecasting techniques relative to the introduction of a new product in a market (diffusion process) will be considered. The conventional diffusion process has been adapted to cope with competition between two operators resorting to Lotka-Volterra like equations. And it will be applied in a specific case study: upgrade of a cellular network from an old to a new technology generation (UMTS to LTE).

6.1 Case study description

Upgrade an access network from an old to a new technology generation (e.g. UMTS to LTE, ADSL to GPON, etc).

6.2 Case study methodology

1. Characterisation of the area of the intervention (geography, demography, main socio-economic activities, existing infrastructures, etc.);
2. Identification of the applicable regulatory framework (licencing conditions, coverage obligations, competition laws, etc.);
3. Identification of the candidate services to be offered as a function of the target users (patterns of life, purchasing power, educational level, etc.);
4. Identification of the candidate technical solutions potentially appropriate to deliver the identified services in the target region;
5. Evaluation of the technical feasibility of the identified solutions (availability, maturity, compliance with standards, etc.);
6. Economic analysis of the identified network solutions;
7. Decision about the direction to follow;
8. Deployment planning;
9. Marketing planning.

6.3 Case study scenarios

The prediction and competition models studied in the previous chapters will now be applied in an economic analysis exercise applied to a specific case: the transition of ADSL technology to GPON in an access network.

The following figure shows a simplified architecture of an ADSL network:

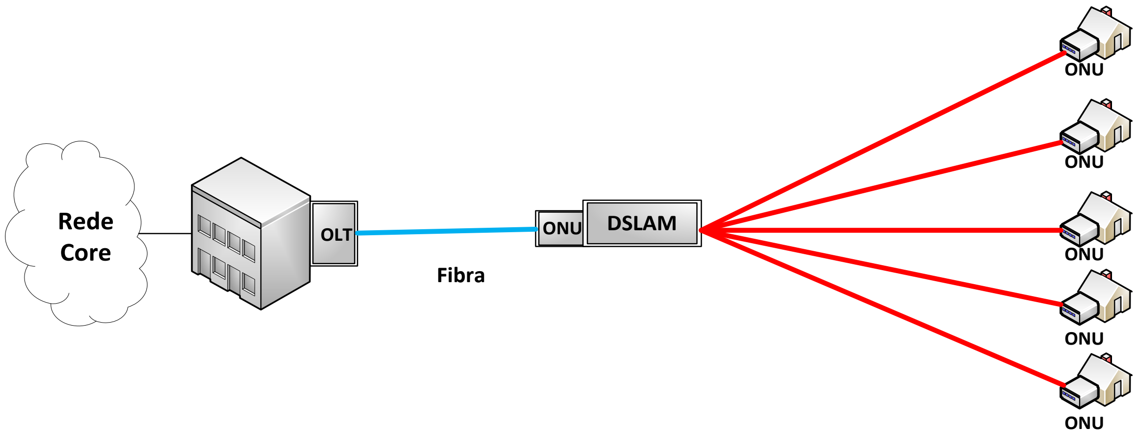

Figure 6-1 Simplified model of ADSL network.

The scenario considered assumes the existence of a legacy infrastructure of copper cables from the telephone network, on which voice and data communication services are provided using ADSL technology.

It should be noted that the simplified model of the above figure assumes the existence of an optical fibre connection between the OLT and the DSLAM.

From this scenario, it is proposed to carry out an economic and financial migration study for GPON technology in an FTTH architecture, the simplified configuration of which is presented in the following figure.

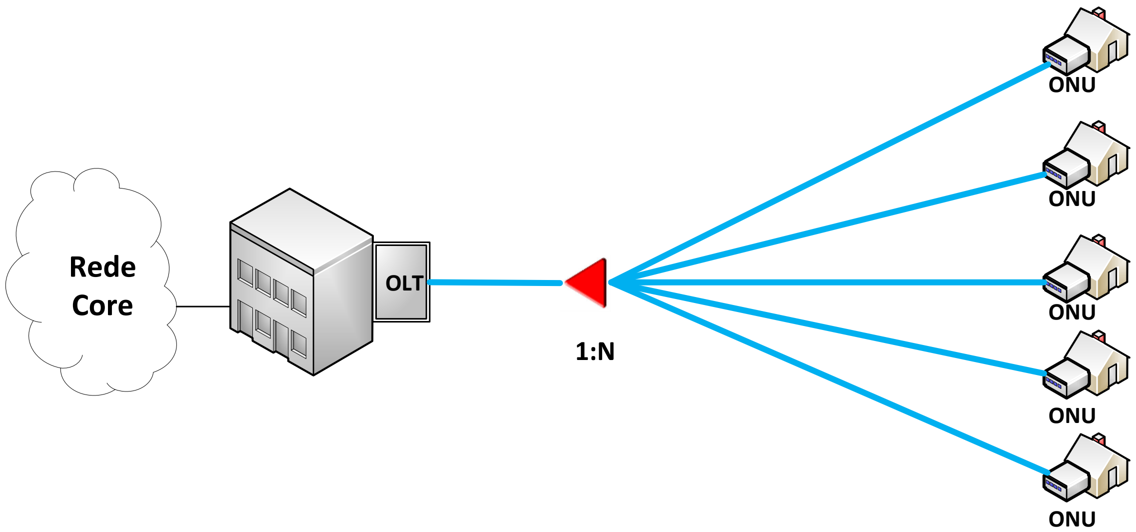

Figure 6-2 Simplified model of a FTTH network.

It is the object of this technological upgrade to take the optical fibre up to the user, installing dedicated fibre to all points of final consumption.

To simplify the case study scenarios that will be consider in this work, the networks architectures that were considered could be represented as the following:

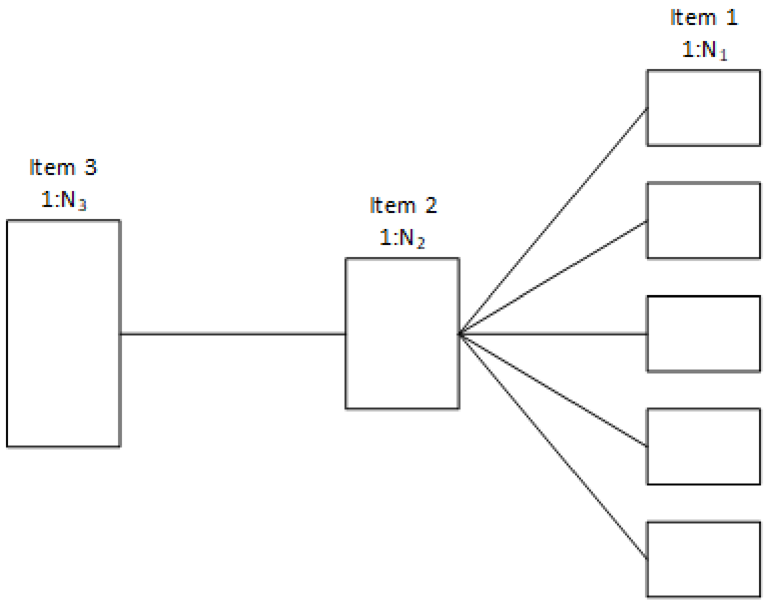

Figure 6-3 Network architecture

Item	Cost (€)	Ratio
1 (User)	C_1	$1: N_1$
2	C_2	$1: N_2$
3	C_3	$1: N_3$

Table 12 Costs and ratios of each item which compose the network.

It is assumed that all costs listed in the above table already include assembly costs and other required work.

The market penetration model of the service is based on the logistics curve equation. This equation can be described by (which has been discussed in Section 3):

$$P = p_i + (p_f - p_i)\frac{1}{1 + \alpha \cdot e^{-\beta \cdot t}} \qquad 31$$

In this formula α and β represent respectively the delay and the growth speed of the logistics curve.

For this dissertation, just one market penetration evolution case was designed based on logistic curve.

The parameters used were:

$Initial\ Penetration\ p_i$	10%
$Final\ Penetration\ p_f$	70%
α	900
β	1.5

Table 13 Parameters used in logistic model for all scenarios.

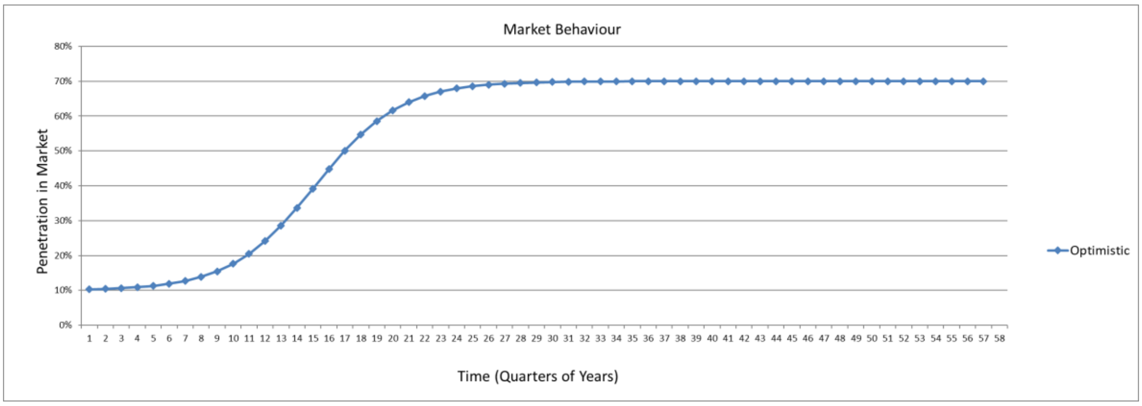

Figure 6-4 Market Dynamics

Assumptions:

To describe the competition between the two operators it was used the following equations (which has been discussed in Section 3):

$$N_1(t+1) = N_1(t) + r_1 N_1(t)\left(1 - \frac{N_1(t) + w_{12}N_2(t)}{K_1}\right)$$

$$N_2(t+1) = N_2(t) + r_2 N_2(t)\left(1 - \frac{w_{21}N_1(t) + N_2(t)}{K_2}\right) \quad 32$$

The growth parameters r_i was 0.5 for both operators and the carrying capacity K_i it was 1, also in both operators. The parameters w_{12} and w_{21} are, as it was seen before, the effect that an operator has on the other. These effects were calculated based in a constant multiplied by a quality function of the respective operator.

To evaluate the quality of a certain operator it was developed the following function:

$$Q_n = (e^{(\sigma \ln\left(\frac{S_n}{\bar{S}}\right) + \beta \ln\left(\frac{TP_n}{\overline{TP}}\right) - \theta \ln\left(\frac{T_n}{\bar{T}}\right))}) \qquad 33$$

In which,

$Q_n = Quality\ of\ Operator\ n$
$S_n = Number\ of\ services\ of\ Operator\ n$
$TP_n = Throughput\ of\ Operator\ n\ (Mbps)$
$T_n = Tariff\ of\ Operator\ n\ (€)$
$\bar{S} = Mean\ number\ of\ services\ of\ the\ market$
$\overline{TP} = Mean\ throughput\ of\ the\ market\ (Mbps)$
$\bar{T} = Mean\ tariff\ of\ the\ market\ (€)$
$\sigma = Weight\ of\ number\ of\ services$
$\beta = Weight\ of\ throughput$
$\theta = Weight\ of\ tariff$

Mean tariff:

$$\bar{T} = \frac{T_1 M_1 + T_2 M_2}{M_1 + M_2} \qquad 34$$

$T_1 = Tariff\ of\ operator\ 1$
$T_2 = Tariff\ of\ operator\ 2$
$M_1 = Market\ share\ of\ operator\ 1\ at\ the\ instant\ t$
$M_2 = Market\ share\ of\ operator\ 2\ at\ the\ instant\ t$

Mean Throughput:

$$\overline{TP} = \frac{TP_1 M_1 + TP_2 M_2}{M_1 + M_2} \qquad 35$$

$TP_1 = Throughput\ of\ operator\ 1$
$TP_2 = Throughput\ of\ operator\ 2$
$M_1 = Market\ share\ of\ operator\ 1\ at\ the\ instant\ t$
$M_2 = Market\ share\ of\ operator\ 2\ at\ the\ instant\ t$

Mean number of services:

$$\bar{S} = \frac{S_1 M_1 + S_2 M_2}{M_1 + M_2} \qquad 36$$

$S_1 = Number\ of\ services\ of\ operator\ 1$
$S_2 = Number\ of\ services\ of\ operator\ 2$
$M_1 = Market\ share\ of\ operator\ 1\ at\ the\ instant\ t$
$M_2 = Market\ share\ of\ operator\ 2\ at\ the\ instant\ t$

The weights of each characteristic were:

Weights	Value
θ	0.4
β	0.3
σ	0.3

Table 14 Weights of each characteristic of the quality function.

Based on the above equations and assumptions it was calculated the quality of each operator. And then the effects of the operators.

$$W_{12} = constant_{12} * Q_{op2}$$
$$W_{21} = constant_{21} * Q_{op1}$$

6.3.1 Fixed Network

In this case it was considered a situation where two network fixed operators coexist in the market. The operator 1 had 80 % of market share and operator 2 20%. A certain time the operator 2, reduces its tariff. It is important to mention that in the fixed network case, if one operator conquers a costumer its competitor must necessarily lose a costumer.

The dimension of the market is 70000 users (a market size like the town of Aveiro).

To this scenario, were considered the following characteristics:

	Operator 1	Operator 2
Tariff (€)	40	20
Throughput (*Mbps*)	100	100
Number of services	3	3
$constant_{12}$	-	1
$constant_{21}$	1	-

Table 15 Characteristics of the operators – fixed network case.

And the following costs:

Item	Cost (€)	Ratio
1 (User)	120	1: 1
2	20 000	1: 1000
3	4 500 000	1: 30

Table 16 Costs of the equipment – fixed network case.

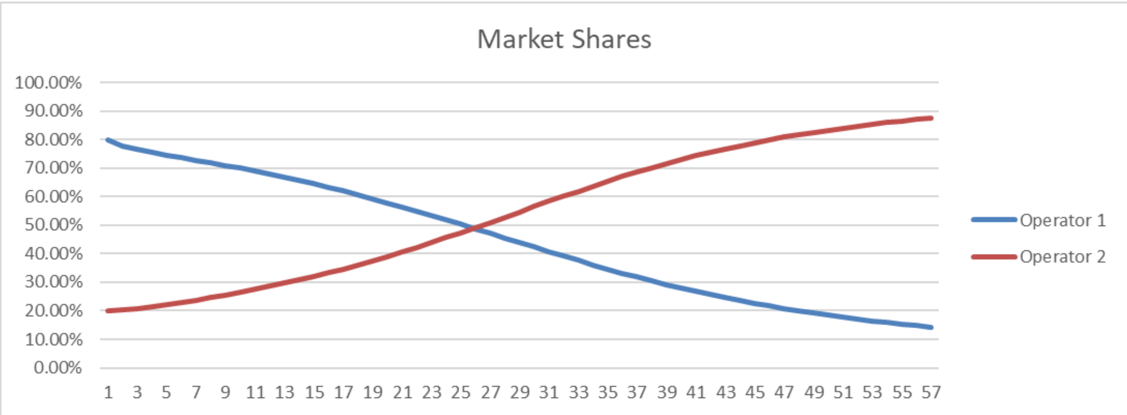

Figure 6-5 Market Shares of each Operator over the time – fixed network case.

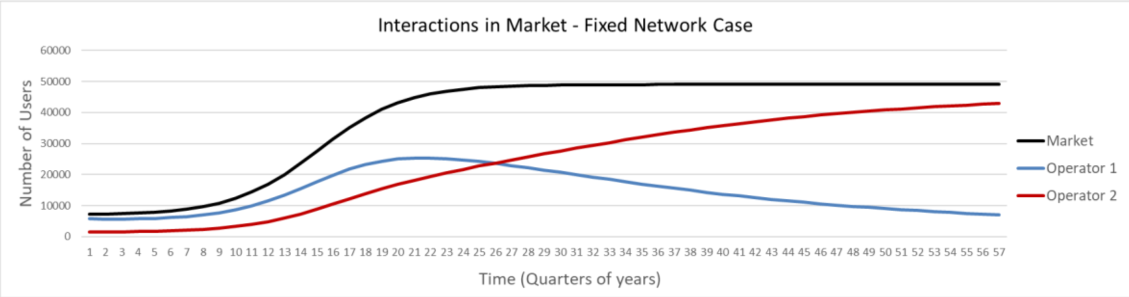

Figure 6-6 Number of users of each operator over the time – fixed network case.

6.3.2 Cellular Network

In this scenario, it was considered a situation where two cellular network operators coexist in the market. The operator 1 had 80 % of market share and operator 2 20%. A certain time the operator 2, reduces its tariff. It is important to mention that in the cellular network case, if one operator its competitor may not lose a costumer. In the case of the cellular network there are often users with more than one terminal.

The dimension of the market is 250000 users.

To this scenario, were considered the following characteristics:

	Operator 1	Operator 2
Tariff (€)	40	30
Throughput (*Mbps*)	100	100
Number of services	3	3
$constant_{12}$	-	0.8
$constant_{21}$	0.8	-

Table 17 Characteristics of each operator – cellular network case.

And the following costs:

Item	Cost (€)	Ratio
1 (User)	120	1: 1
2	20 000	1: 1000
3	5 000 000	1: 30

Table 18 Costs of the equipment – cellular network case.

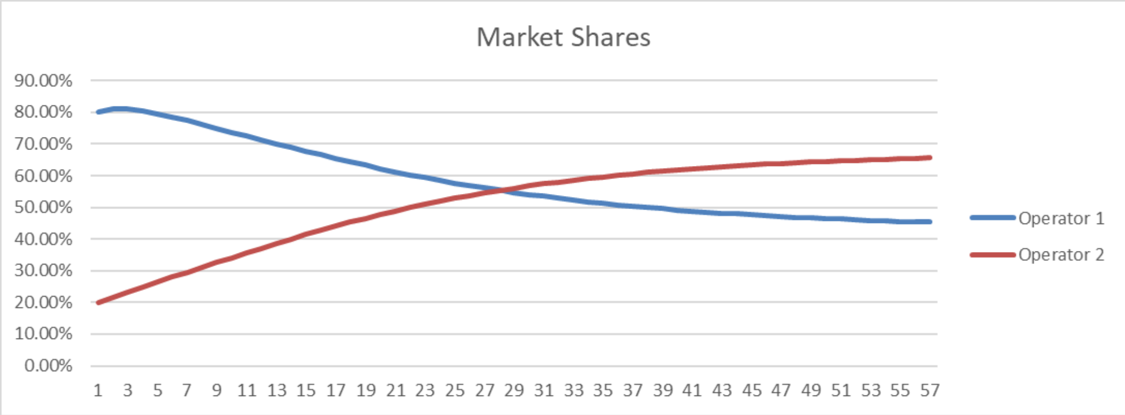

Figure 6-7 Market share of each operator over the time – cellular network case.

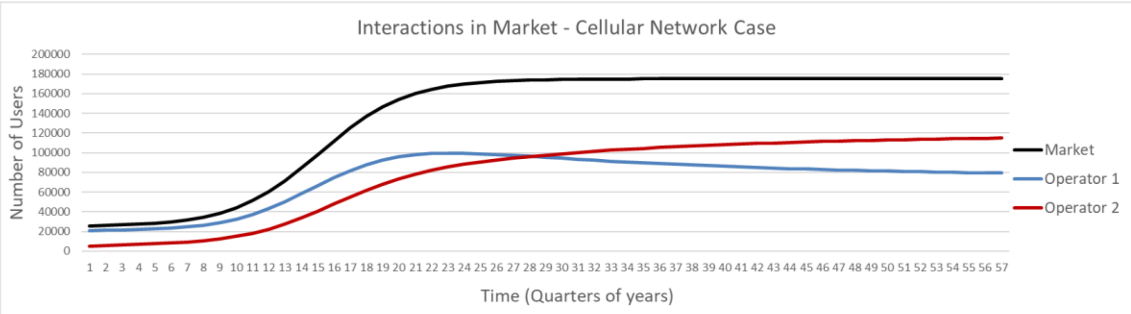

Figure 6-8 Number of users of each operator over the time – cellular network case.

With these results, it was possible to made a techno-economic analysis for the two case study scenarios.

6.3.3 CAPEX

CAPEX (Capital Expenditure) reflects all the investment necessary to implement the network infrastructure, that includes all the expenses incurred in the purchase, improvement or extension of the life of the tangible fixed assets.

The investment needs over the life of the project is determined by:

$$Capex(t) = \sum_{i=0}^{N} Capex_i(t) \qquad 37$$

Reflecting the sum of CAPEX for each of the items considered.

The CAPEX of each item reflects the amount of materials and equipment installed, determined by the evolution of the number of users over time and by the ratio associated with the item under analysis.

It should be noted that the initial investment is considered materialized on "instant 0", that is, before the beginning of the economic activity. At that moment, the necessary investment is made to start the project, so that in the first year of the economic year the service can be provided to the intended customers (according to the initial defined penetration rate).

According to the previously described it was possible calculate the CAPEX of the two cases.

Fixed network case:

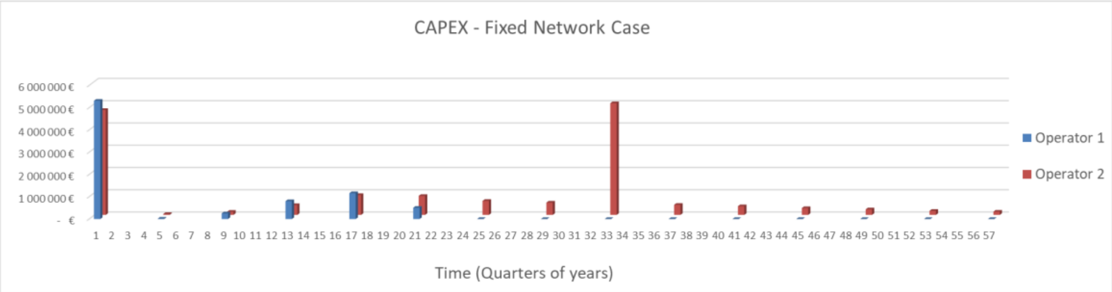

Figure 6-9 CAPEX of the two operators – fixed network case.

Cellular network case:

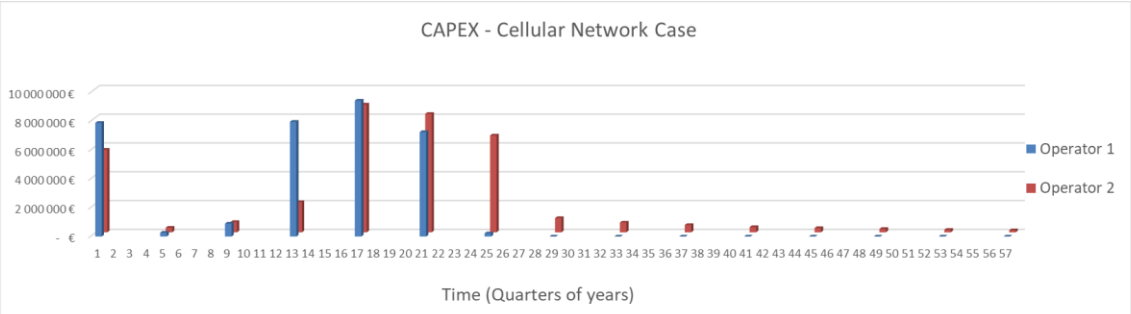

Figure 6-10 CAPEX for each operator – cellular network case.

6.3.4 OPEX

OPEX (Operational Expenditure) reflects all the investment required to maintain the network infrastructure, that includes all operating expenses over the life of the network.

OPEX is not intended to increase the assets, that is, the assets that have the capacity to contribute to the generation of future income of an institution. All costs or expenses necessary for the normal functioning of an entity are OPEX. These costs can be: Expenses with personnel, services acquired from others (e.g. maintenance of equipment, marketing, etc.), raw materials, periodic licensing with a term of less than 1 year, financial costs (in some cases), among others.

In this work, the OPEX is calculated by a percentage of 15% of the CAPEX of each case and a fixed value per user (costumer) which is 150 € in both cases.

According to the previously described it was possible calculate the OPEX of the two cases.

Fixed network case:

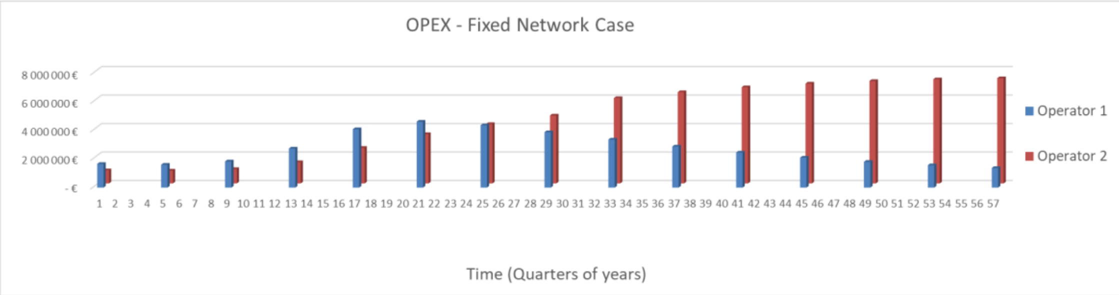

Figure 6-11 OPEX for Two Operators – fixed network case.

Cellular network case:

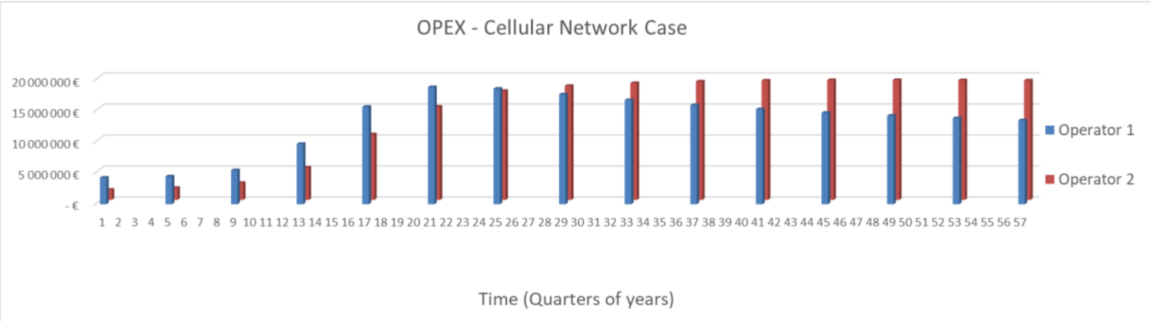

Figure 6-12 OPEX for each operator – cellular network case.

6.3.5 Revenues

The revenues (or profits) and gains includes all the incomes that give rise to positive cash flows, that is, the instalments to into cash:

- Operating revenues (sales of goods, provision services, etc.);
- Financial revenues (income from financial investments, for example);
- Extraordinary gains arising from any another situation not contemplated above.

To calculate the revenues, it was necessary to be in consideration the dimension of the market in each case, that is, the number of potential users (costumers) and the penetration rate of the offer services.

For both cases (fixed and cellular), it was considered an annual cost of the product of 420 €, and an erosion rate of 2 %.

Fixed network case:

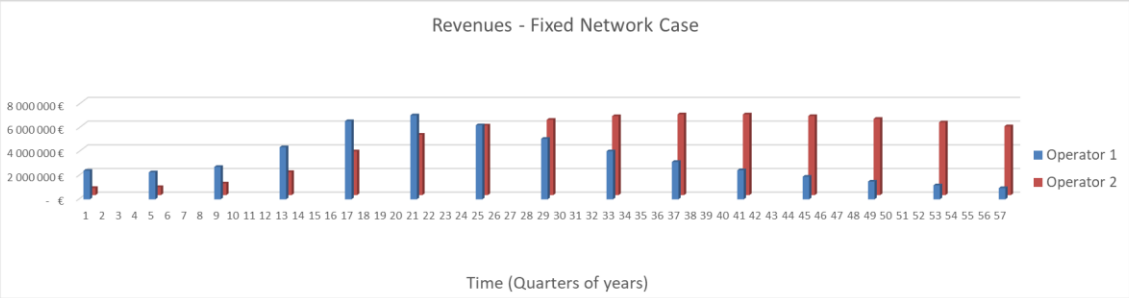

Figure 6-13 Revenues for Two Operators – fixed network case.

Cellular network case:

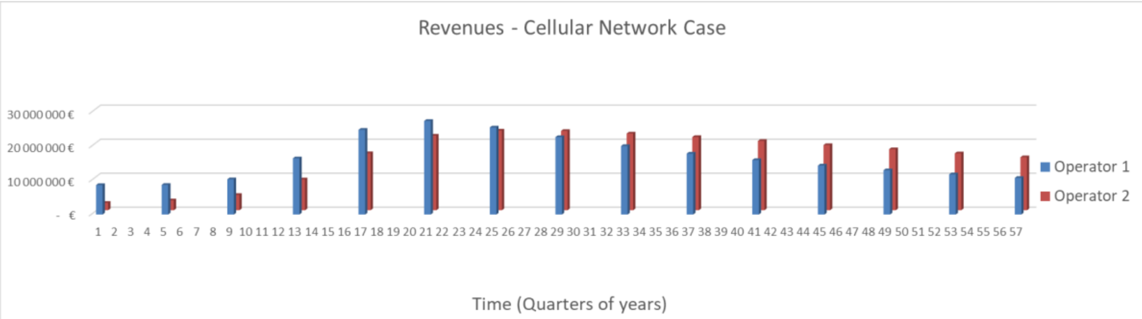

Figure 6-14 Revenues of each operator – cellular network case.

6.4 Results Discussion

In this subsection, first it will be present some methods of evaluating projects. Then according to these methods, it will be discussed the economic results of the two scenarios studied before.

Methods of evaluating projects

When comparing, adding or subtracting expenses, revenues or cash flows carried out on different dates, it is necessary to first refer them to the same date, often the start date of the project, and update them at a given annual rate known as reference rate or discount rate. Thus, the present value of a future income or expense. [28]

NPV – Net Present Value

The NPV is the sum of all cashflows, taking in account the discount rate r and corresponding to t intervals of time, of expenses E_p and revenues R_p (with p from 1 until t), that occurs p (years) dated of the initial instant, is given by: [28]

$$NPV = \sum_{p=1}^{n} \frac{R_p - E_p}{(1+r)^p} - I \qquad (38)$$

Where

$R_p = revenues\ during\ period\ p$
$I = total\ initial\ investment\ costs$
$E_p = expenses\ during\ period\ p$
$r = discount\ rate$

If the Net Present Value is greater than 0, it means that the gains generated by the project or investment outweigh its own costs. In most cases an investment or project with positive NPV is profitable, and with negative NPV it is a failure. This metric is reinforced by the Net Present Value Rule, which says that the only investments to be wagered are those with a positive NPV. [29]

Payback Period

An alternative metric to net present value is the Payback Period. Unlike the first metric, the recovery period does not account for the value of money in time. Hence, the return periods calculated for long-term investments are more imprecise because they encompass more time during which inflation can occur and distort projected earnings and as well as the actual period of recovery. This metric is much simpler than the NPV, focusing on the assessment time needed after an investment to recover the initial costs of that investment. [29]

This method consists only of the period that the investment project takes to recover the investment, that is, the period that elapses until the cash flows from negative to positive refers to the recovery period. Although the calculation is simple, it does not consider what happens after the recovery period, nor does it allow to assess the profitability of the project. [28]

Internal rate of return (IRR)

Another alternative to NPV is the internal rate of return (IRR). IIR is calculated in the same way as NPV, but with some differences. The IRR assumes a neutral NPV (a value of zero) and instead resolves the discount rate. The IRR is annual - refers to projected returns annually - this metric facilitates the comparison, in a simple way, of a wide range of investments. [29]

6.4.1 Fixed Network

Operator 1:

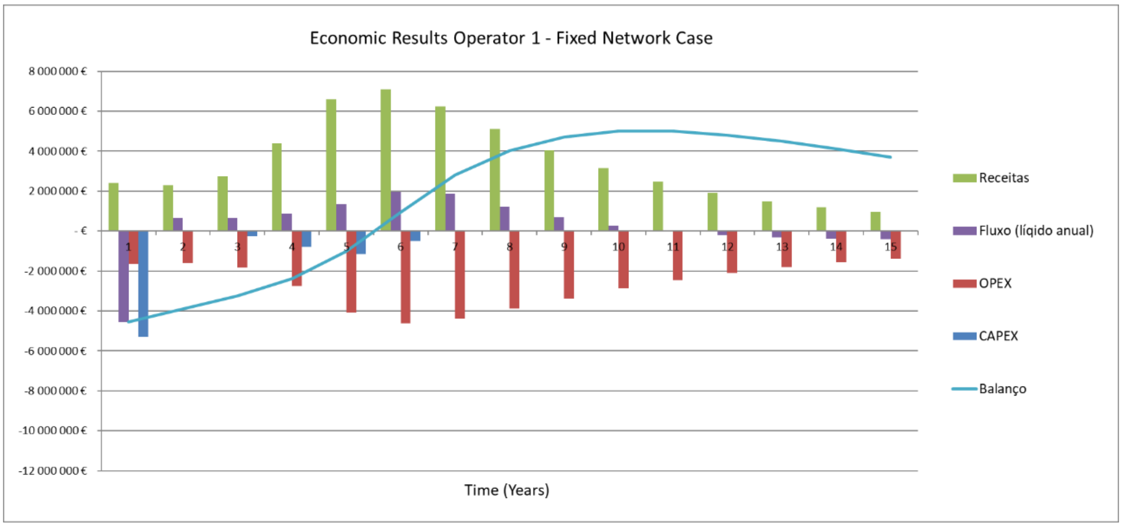

Figure 6-15 Economic results Operator 1 – fixed network case.

Revenues grow until the sixth year, and then begin to decline rapidly, due to the loss of market share that occurs from that year.

CAPEX is more pronounced in the fourth, fifth, and sixth years, and ceases to exist in subsequent years. Since the operator is losing market share during this period and therefore its network infrastructure is sufficient for the customers it has.

OPEX also declines from the sixth year again due to the decline in market share.

However, the balance is negative until the middle of the fifth year becoming positive to move from that date. Enter the operator in the payback period.

In terms of market indicators Operator 1 presents the following:

Market Indicators – Operator 1 – Fixed Network	
NPV (€)	2 169 068.45 €
IRR (%)	16.15%
Payback Period (Years)	5

Table 19 Market Indicators – Operator 1 – Fixed Network

The NPV is positive just it was expected, and the IRR is acceptable. The payback period is 5 years. These parameters show that the project of this operator is viable.

Operator 2:

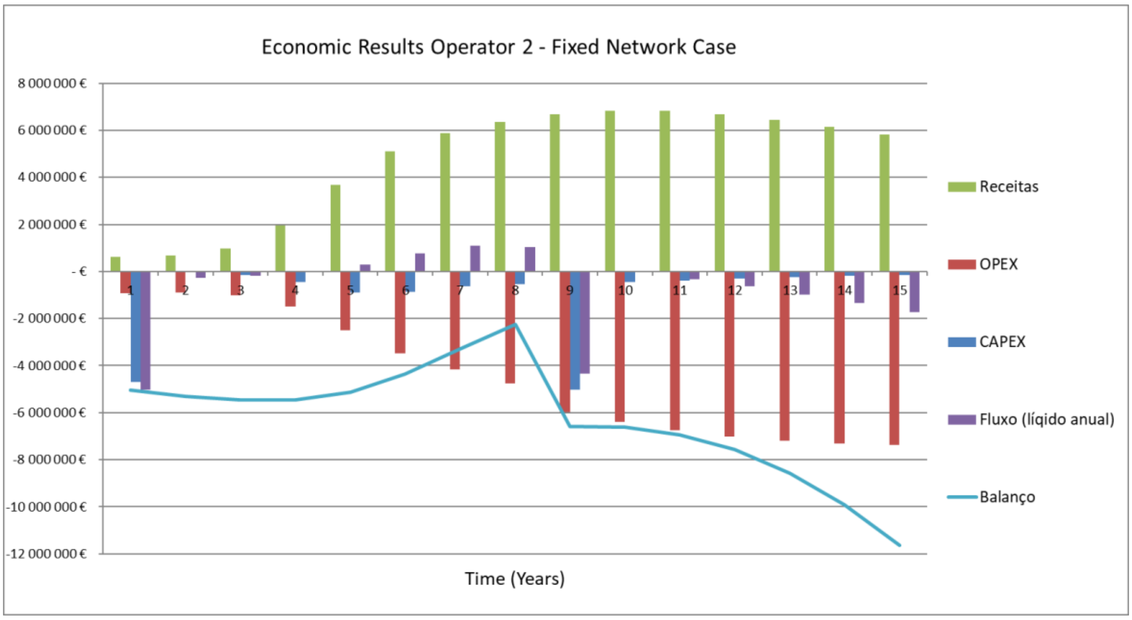

Figure 6-16 Economic results Operator 2 – Fixed network case.

Revenues grow at a parallel pace to the trend of customer adherence to the operator. The fact that there are more customers joining Operator 2 also implies that the latter has more expenses.

In terms of CAPEX there is a great investment in the first year and in the ninth year. In the first year the installation of the necessary network infrastructure for the start of operation is considered. In the ninth year, an investment in network infrastructure is again made, which means that the existing infrastructure is no longer sufficient to serve all the operator's customers.

As a result, OPEX also increases. As the infrastructure has grown and the number of customers has also increased over time the costs associated with them also increase.

Although the operator's revenues have a growing trend, expenses also increase. This is reflected in the balance sheet, which is negative throughout the project period. Up to the eighth year there may be a growing trend, albeit in negative territory, but with the investment of infrastructure in the ninth year the balance falls to an even more critical area. Never reaching the payback period.

Given this, the market indicators of this operator are:

Market Indicators – Operator 2 – Fixed Network	
NPV (€)	-8 264 721.65 €
IRR (%)	-
Payback Period (Years)	15

Table 20 Market Indicators – Operator 2 – Fixed Network

The NPV is negative just it was expected. The payback period is 15 years. These parameters show that the project of this operator is not viable.

6.4.2 Cellular Network

Operator 1:

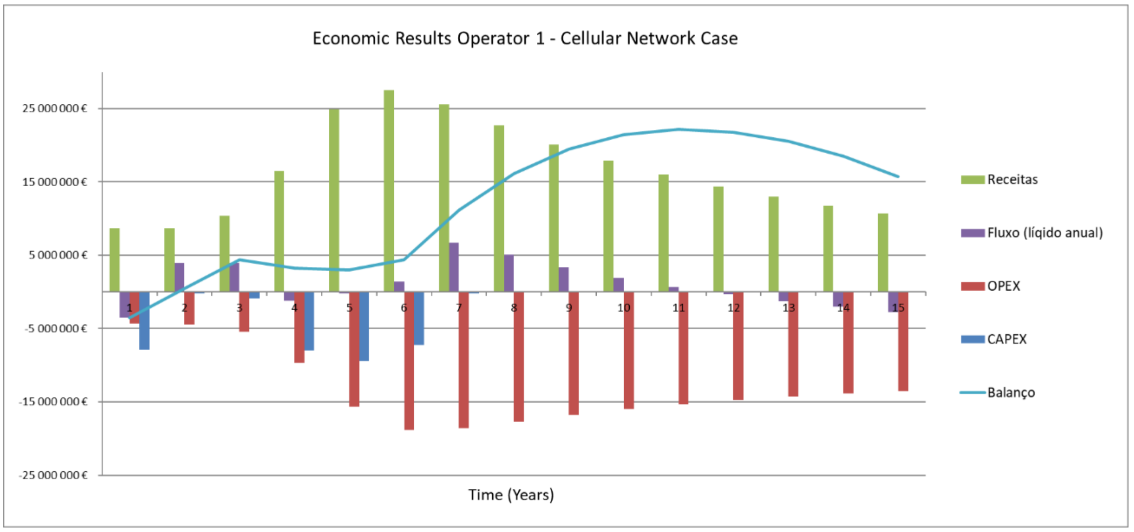

Figure 6-17 Economic results Operator 1 – Cellular network case.

Revenues grow at an accelerated pace through the sixth year. From then on, a gradual descent.

An investment in infrastructure is made in the first year of the project, as well as in the fourth, fifth and sixth. These investments are reflected in terms of CAPEX in those same years.

In terms of the market share of this operator, there is a sharp initial rise, followed by a slight decline and then stabilized. This can be seen in OPEX, where expenditure rises rapidly up to the sixth year, falling slightly over the next two years and stabilizing thereafter.

This operator reaches the payback period shortly after the second year of the project. In the third year there is a slight decrease in the balance sheet that extends up to the sixth year and is due to the investment made in infrastructure in this period. From the sixth to the eleventh year, the balance returns to increase, starting to decrease again, starting from this year, which is mainly due to the erosion rate of the product.

The market indicators of this operator are presented in the following table:

Market Indicators – Operator 1 – Cellular Network	
NPV (€)	12 281 396.65 €
IRR (%)	82.32%
Payback Period (Years)	1

Table 21 Market Indicators – Operator 1 – Cellular Network

The NPV is positive just it was expected, and the IRR is very acceptable. The payback period is 1 year. These parameters show that the project of this operator is viable.

Operator 2:

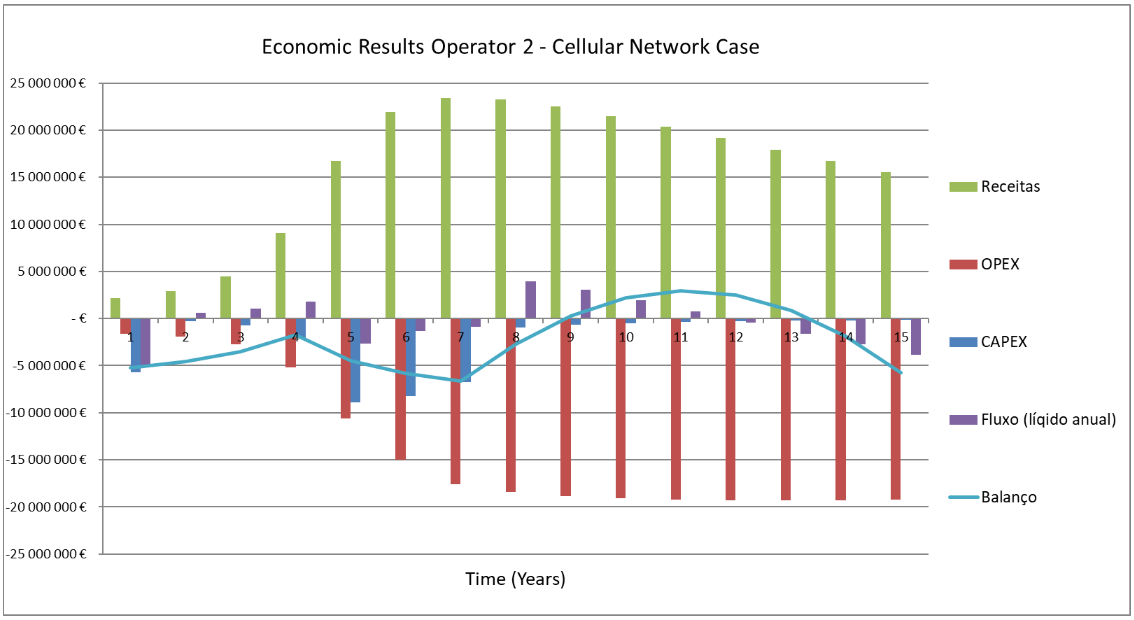

Figure 6-18 Economic results Operator 2 – Cellular network case.

The revenues of this operator grow to the eighth year, sharply, beginning to decline form downwards.

It is possible to verify, through the CAPEX values, an investment in network infrastructure in the first year of operation, and later in the fifth, sixth and seventh years. This is due to a gradual increase in market share during this period.

This increase in market share is reflected substantially in OPEX, which grows sharply up to the eighth-year stabilizing thereafter.

Finally, the balance sheet is in negative territory until the ninth year. Despite a growing trend in the first four years, there is a drop that is due to investments in infrastructure. Growing back from the seventh to the eleventh year.
From the ninth year until the middle of the thirteenth, this operator enters a payback period and then returns to negative territory.

The market indicators for this operator are as follows:

Market Indicators – Operator 2 – Cellular Network	
NPV (€)	-3 884 108.83 €
IRR (%)	-
Payback Period (Years)	8

Table 22 Market Indicators – Operator 2 – Cellular Network

The NPV is negative just it was expected. The payback period is 8 years. These parameters show that the project of this operator is not viable.

7. Conclusion and future work

This work provided familiarization with the different forecasting techniques in the context of telecommunications planning. Several methods were studied and their main advantages and disadvantages identified. These methods were applied in telecommunication case study scenario.

Another objective of this work was to understand how to deal with competition and substitution effects in telecommunications markets. Resorting to modified Lotka-Volterra equations it was possible create useful models and apply them to a techno-economic analysis to understand the impact that different strategies can have on inter operator competitiveness.

In terms of future work, several challenges remain:

- Test more methods of forecast to improve the methodology of the decomposition of time series;
- Develop and test accuracy measures.
- Improve the quality function presented in this work, in order to reflect all the variables that have influence in the quality of the operator in cause.

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